East African Community (EAC)

Federation of East African Freight Forwarders Associations (FEAFFA)

THE EAST AFRICA CUSTOMS AND FREIGHT FORWARDING PRACTICING CERTIFICATE

# NUMERACY SKILLS

FEAFFA in collaboration with East Africa Revenue Authorities

**East African Community (EAC)**

The East African Community (EAC) is a regional intergovernmental organization of six (6) Partner States, comprising Burundi, Kenya, Rwanda, South Sudan, Tanzania and Uganda, with its headquarters in Arusha, Tanzania.

**Federation of East African Freight Forwarders Associations (FEAFFA)**

The Federation of East African Freight Forwarders Associations (FEAFFA) is a regional private sector apex body of the Customs Clearing and Freight Forwarding (CFA) industry in East Africa. It aims at promoting a professional freight logistics industry for trade facilitation and regional economic growth. FEAFFA strives to address the challenges experienced by its members through training, provision of information, and other aspects of capacity building. It advocates for the full implementation of the East African Community (EAC) Customs Union. The East Africa Customs and Freight Forwarding Practicing Certificate (EACFFPC) is the Federation's and the industry's premier training program in East Africa since 2007.


Copies may be obtained through the following address:

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# FOREWORD

Customs Clearing Agents, Freight forwarders, and Warehouse Operators in the East African Community (EAC) region continue to play a vital role in the facilitation of trade particularly with regards to the assessment tax, storage of goods, transportation, and last-mile delivery to clients. This, in turn, facilitates cargo movement and clearance from all ports.

The agents handle goods worth millions of dollars on behalf of the shippers. Besides, they originate documents that facilitate movement and clearance of cargo culminating in errors that slow down the flow of business. The movement of cargo depends on how fast and correctly documentation is done for verification by the respective Customs Authorities. A delay in customs clearance increases the cost of doing business.

This pointed to the need for these agents to be equipped with the requisite knowledge, skills, and attitudes to carry out their work efficiently, just as their counterparts from customs.

The EAC region, with support from TradeMark East Africa (TMEA), has made significant steps towards bridging the knowledge and skills gap in the customs clearing and freight forwarding industry. The introduction of the East Africa Customs and Freight Forwarding Practicing Certificate (EACFFPC) in 2006, a regional training programme jointly implemented by the EAC directorate of customs, the East African Revenue Authorities (EARAs), the National Association of the Freight Forwarding Industry, and FEAFFA was a big step. Since its inception, over 7000 agents have graduated from this training.

A review of the programme in 2015 and a market survey conducted in 2020 supported by TradeMark East Africa (TMEA) highlighted key areas of improvement for the EACFFPC programme to achieve the aim of producing competent customs agents, freight forwarders, and warehouse keepers. The revised curriculum has therefore been designed to address these challenges and shortcomings. The revamped EACPPFC programme is designed to enhance the ability of freight forwarders to provide competitive and high-quality end-to-end services thereby reducing inventory costs and increasing safety levels in warehousing operations in the East African region.

With the revised EACFFPC curriculum, the dream of attaining a professional and compliant freight logistics industry in the East African region has been strongly boasted.

# ACKNOWLEDGMENT

The Curriculum Implementation Committee (CIC) is grateful to the EAC sectoral council on Trade Industry Finance and Investment for adopting the EACFFPC as an EAC training programme for clearing and forwarding agents in the region. This is a testimony to the effect the programmeme has had on the clearing and forwarding industry in the EAC region.

The CIC is also grateful to the EAC Directorate of Customs, the Commissioners of Customs of the East Africa Revenue Authorities, the Chairpersons of National Associations of clearing and forwarding agents, and the President of FEAFFA for their dedication and support to the EACFFPC programmeme.

Special appreciation for the National Curriculum Implementation Committees for providing their trainers to participate in the development and validation of the curriculum and training materials. CIC also acknowledges the FEAFFA secretariat for excellently coordinating the curriculum and training materials development and validation process.

The CIC in a very special way recognizes TradeMark East Africa (TMEA) who provided the financial support to update the curriculum, develop and publish the 2021 edition of the EACFFPC training materials. We remain indebted to you forever.

We also appreciate all EACFFPC trainers, students, and stakeholders for the constant feedback that has been incorporated in this edition of the training materials.

# NUMERACY SKILLS

# 1.0 UNIT OVERVIEW

## 1.1 Unit Description

This unit covers the competencies required to perform numerical functions. The person who is competent in this unit shall be able to: Calculate with whole numbers and familiar fractions, decimals and percentages for work; Estimate, measure, and calculate with routine metric measurements for work; Use routine maps and plans for work; Interpret, draw and construct 2D and 3D shapes for work; Interpret routine tables, graphs and charts for work; Collect data and construct routine tables and graphs for work; and Use basic functions of calculator.

## 1.2 Unit Summary Learning Outcomes

At the end of the unit, the trainee should be able to:

1. Calculate with whole numbers and familiar fractions, decimals and percentages for work
2. Estimate, measure and calculate with routine metric measurements for work
3. Use routine maps and plans for work
4. Interpret, draw and construct 2D and 3D shapes for work
5. Interpret routine tables, graphs and charts for work
6. Collect data and construct routine tables and graphs for work
7. Use basic functions of calculator

# 2.0 WHOLE NUMBERS AND FAMILIAR FRACTIONS, DECIMALS AND PERCENTAGES FOR WORK

## 2.1 Specific Learning Outcomes

At the end of the topic, the trainee should be able to:

i. Interpret of whole numbers, fractions, decimals, percentages and rates
ii. Calculate involving several steps
iii. Calculate with whole numbers and routine or familiar fractions, decimals and percentages
iv. Converse between equivalent forms of fractions, decimals and percentages
v. Apply order of operations to solve multi-step calculations
vi. Apply problem-solving strategies
vii. Make estimations to check reasonableness of problem-solving process, outcome and its appropriateness to the context and task
viii. Use of formal and informal mathematical language and symbolism to communicate the result of a task

## 2.2 Whole Numbers

### 2.2.1 Overview of Whole Numbers

**Whole number** are digits 0,1,2,3,4,5, 6,…….

There are **counting numbers** which are digits 1,2,3, 4,…… Note that this does not include zero.

Both Whole numbers and Counting Numbers are referred to as **Natural numbers**

### 2.2.2 Operation on Whole Numbers

**Addition**

21+30=51

**Subtraction**

2753 – 376=2,377

**Multiplication**

i) A health centre distributes 2 689 bottles of hand washing soap every month. How many bottles does the health centre distribute in 3 years?

```
  2 689
 X   36
 16134 (2689 X 6)
 80670 (2689 X 30)
 96804
```

**Division**

Work out : 1769 ÷ 32

```
        55
32 | 1769
     -160
      169
     -160
        9
```

Here 55 is the quotient and 9 is the remainder.

```
        29
124 | 3654
     -248
      1174
     -1116
        58
```

### Digital Corner

Using a digital device open the link; [https://www.iknowit.com/lessons/d-division-3-digit-dividends-with-3-digit-quotients.html](https://www.iknowit.com/lessons/d-division-3-digit-dividends-with-3-digit-quotients.html). Work out division exercises on the link. The trainee should click on "Hint" Button to display various division methodologies so as to effectively acquire the competence of division operation.

## 2.3 Fraction

By dividing a whole, a portion of the whole is called a fraction. Consider an orange which is cut into four equal parts. Each piece of the whole is called a Fourth/quarter i.e. $\frac{1}{4}$. Therefore, is a fraction where $a$, and $b$, are numbers. $a$ which is the number on top and indicate the number of parts is called numerator and $b$ which is the number at the bottom and show the numbers of parts the whole is divided into is called denominator

Fractions can be classified into categories:

i) **Proper fraction:** Under proper fraction, the numerator of the fraction is smaller than the denominator of the fraction. Examples of proper fractions are among others.

ii) **Improper fraction:** A fraction where numerator is larger than the denominator is referred to as improper fraction. Examples of improper fractions include among others.

iii) **Mixed fraction or mixed number:** A Mixed fraction comprise of a whole number and a fractional part. Examples of mixed fractions include 3 , 1, 18 among others.

In order to compare fractions in terms of their magnitude, one has to obtain Lowest Common Multiple for the denominators as given.

### Comparing fractions

Use illustration of a rectangle divided into 2, 3 and 4 parts and shade a part of the numerator to show comparison where;

$\frac{1}{2} = \frac{3}{6} = \frac{4}{8}$

From the illustration you will realize that fractions $\frac{1}{2}$, $\frac{3}{6}$, and $\frac{4}{8}$ are equivalent fractions.

### Example.

Which of the fractions, $\frac{2}{3}$ and $\frac{3}{4}$ are greater?

Equivalent fraction $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$ , therefore is greater than $\frac{8}{12}$ and so $\frac{3}{4}$ is greater than $\frac{2}{3}$.

### Arranging Fractions in Order; descending or Ascending

### Operation On Fraction

#### i) Addition of mixed fractions

Example 1

Work out the following fractions.

$2\frac{3}{5} + 4\frac{3}{4}$

$2\frac{3}{5} + 4\frac{3}{4} = \frac{13}{5} + \frac{19}{4}$ (Convert the fractions into improper fractions)

$= \frac{52+95}{20}$ (Determine the LCM of 5 and 4 and add improper fraction)

$= \frac{147}{20}$

$= 7\frac{7}{20}$

$2\frac{3}{5} + 4\frac{3}{4} = 7\frac{7}{20}$

Zawadi bought $5\frac{4}{5}$ of oranges and $4\frac{1}{4}$ of mangoes. What is the total mass of the fruits bought?

$5\frac{4}{5} + 4\frac{1}{4} = (5 + 4) + (5\frac{4}{5} + 4\frac{1}{4})$ (Add whole numbers)

$= 9 + \frac{4}{5} + \frac{1}{4}$

$= 9 + \frac{16+5}{20}$

$= 9 + 1\frac{1}{20}$

$= 10 + \frac{1}{20}$

#### ii) Subtraction of mixed fractions

Work out the following fractions.

Subtract $3\frac{1}{2}$ from $5\frac{2}{3}$ .

$5\frac{2}{3} - 3\frac{1}{2} = \frac{17}{3} - \frac{7}{2}$ (Convert fraction into improper fractions)

$= \frac{34-21}{6}$ (Subtract the improper fraction using LCM of denominators)

$= 13/6$

$= 2\frac{1}{6}$

### Multiplication and Division of Fractions

Work out the following fractions.

Example 1

$3\frac{3}{7} \times 2\frac{4}{5} = \frac{24}{7} \times \frac{14}{5}$
$= 48/5$
$= 9\frac{3}{5}$

### Example 2

i) Simplify $6\frac{1}{3} + 5\frac{1}{9} - 2\frac{2}{7}$

Solution:
The whole number part = 6 + 5 – 2 = 9

Considering the fractions

$\frac{1}{3}+\frac{1}{9}-\frac{2}{7}$

The LCM of 3, 9 and 7 is 63

For fractional parts we have $\frac{21}{63}+\frac{7}{63}-\frac{18}{63} = \frac{10}{63}$

Now, combining the whole number part, we have
$9 + \frac{10}{63} = 9\frac{10}{63}$

Alternatively;

$6\frac{1}{3} + 5\frac{1}{9} - 2\frac{2}{7} = 6 + 5 - 2 + (\frac{1}{3}+\frac{1}{9}-\frac{2}{7})$

$= 9 + \frac{(21+7-18)}{63}$

$= 9 + \frac{10}{63}$

$= 9\frac{10}{63}$

### Order of operations on fractions (Combined Operations)

1. Evaluate.

$$\frac{1}{2}\left(\frac{3}{5}+\frac{1}{4}\left(\frac{7}{3}-\frac{3}{7}\right) of\ 1\frac{1}{2} \div 5\right)$$

Use Bodmas to solve.

$= \frac{1}{2}\left(\frac{3}{5}+\frac{1}{4}\left(\frac{40}{21}\right) of 1\frac{1}{2} \div 5\right)$

$= \frac{1}{2}\left(\frac{3}{5}+\frac{1}{4}\times\frac{40}{21}\times\frac{3}{2} \div 5\right)$

$=\frac{1}{2}\left(\frac{3}{5}+\frac{10}{21}\times\frac{3}{2} \div 5\right)$

$=\frac{1}{2}\left(\frac{3}{7}+\frac{5}{7} \div 5\right)$

$=\frac{1}{2}\left(\frac{3}{5}+\frac{5}{35}\right)$

$=\frac{1}{2}\left(\frac{21+5}{35}\right)$

$=\frac{1}{2}\times\frac{26}{35}$

### Digital Corner

Use digital device to open the link https://www.mathgames.com/skill/5.87-multiply-a-mixed-number-by-a-fraction. Play the digital games on fractions. The trainee can select any other relevant digital content outlined in the syllabus and practise on operation on fractions.

## 2.4 Decimals

A fraction whole denominator is written as power to 10 is called a decimal fraction. Consider $3/10$, $4/100$, $17/1000$. These all are decimal fractions and they have a special way of been written i.e. 0.3, 0.04, 0.017 which are simply referred to as **Decimals**

The Dot in the notation is the **decimal point** such that the number is said to be **Zero point three.**

**Then,** if a number is 8.3, it means $8 + 3/10$

## Conversion of Fractions and Decimals

Convert fractions into decimals

### Example 1

Express each of the following fractions into decimals.

1. $\frac{5}{10} = 0.5$
2. $\frac{6}{20} = \frac{6 \times 5}{20 \times 5} = \frac{30}{100} = 0.3$

## Conversion of decimals into Fractions

1. $0.04 = \frac{0.04}{1} \times \frac{100}{100} = \frac{4}{100}$
2. $0.314 = \frac{3}{10} + \frac{1}{100} + \frac{4}{100}$

$= \frac{300}{1000} + \frac{10}{1000} + \frac{4}{1000}$

$= \frac{314}{1000}$

## Meaning of decimals

Decimals are fractions, which have denominators of 10, 100 and so on, according to the position of the figure after the decimal point. The dot, also called the decimal point, separates the whole numbers from the fractional parts. The importance of the decimal point is noted whenever exact placing of numbers is required for addition.

For instance 667.67 = 600 + 60 + 7 + $\frac{6}{10} + \frac{7}{100}$

## Operations on decimals

Addition and subtraction of decimals is performed the same way whole numbers are manipulated. Addition and subtraction are carried out correctly when the digits involved are arranged according to their place values.

### Example:

i) Evaluate 7.934 + 3.208 + 2.6 – 1.425

*Solution:*

Given the problem 7.934 + 3.208 + 2.6 – 1.425

Consider the addition first

Then,

$$\begin{array}{r} 7.934 \\ +\ 3.208 \\ \underline{2.6} \\ 13.742 \\ \underline{-\ 1.425} \\ \underline{\underline{12.317}} \end{array}$$

## Recurring Decimals

Sometimes in division, the result may be a number with a digit or a group of digits repeating continuously without ending. Such decimal fractions are called recurring decimals.

Example:

Work out  and

The answers are 0.33333… and 0.454545….

You have realized that in the first case the digit 3 is recurring and in the second case digits 45 are recurring.

In mathematics then we indicate recurring by putting a dot on the top of the recurring digit(s) or in the first and the last digit in the pattern.

Consider

= 0.324324324……. in this case the recurring numbers are 324. So the dot is placed on top of 3 and 4.

## Decimal place

By dividing a number, the process may go on and on without ending. In such a case, you may be required to round off the answer. Rounding off is done to the require digit on the right side of decimal. This is called **decimal place.**

### Example

1.5  1.3 =1.15384615…..

The answer may be rounded off to

i) And so on 1.2 to the nearest tenth (1 decimal place)
ii) 1.15 to the nearest hundredth (2 decimal place)
iii) 1.154 to the nearest thousandth (3 decimal place)

## Converting Recurring Decimals to Fractions

### Steps in Converting Recurring Decimals to Fractions

i) Let x = Recurring decimal
ii) Let n = The recurring digits
iii) Multiply the recurring decimal by $10^n$
iv) Subtract (1) from (3) to eliminate the recurring part
v) Solve for x, expressing your answer as a fraction in its simplest form

### Example 1

Convert 0. to fraction form.
Let x = 0.7777
10x = 7.777
10x – x = 7.7777 – 0.7777
9x = 7
x =

### Example 2

Convert 3. to fraction form.
Let x = 3.2727
Let 100x = 327.2727
100x – x = 327.2727 - 3.2727
99x = 324
x = $^{324}/_{99}$
= $^{36}/_{11}$

### Standard form

If a number is expressed in form of A $x10^n$ then, this called standard form.

### Example

Write the numbers in standard form.

I. 36
36 = $^{36}/_{10}$ x10 = $3.6x10^1$

II. 0.052

0.052 = 0.052 x $^{100}/_{100}$
= 5.2 x $^{1}/_{100}$
= 5.2 x $(^{1}/_{10})^2$

= 5.2 x $10^{-2}$

### Operation on decimals

### Example 1

Work out: 117.1271 + 122.3913
Figure 1

| Hun-dred | Tens | Ones | Dec-imal Point | Tenths | Hun-dredths | Thou-sandths | Ten Thou-sandths |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 7 | . | 1 | 2 | 7 | 1 |
| + 1 | 2 | 2 | . | 3 | 9 | 1 | 3 |
| 2 | 3 | 9 | . | 5 | 1 | 8 | 4 |

Application of Whole Numbers in problem-solving process, outcome and its appropriateness to the context and task

## 2.5 Percentages

This is simply a part of 100. Normally written are %. Consider a student score of. Then the percent is 60%.

### Converting Fractions and Decimals into percentages

### Example

Change $\frac{2}{5}$ into percentage

$\frac{2}{5} = \frac{*}{100}$

$* = \frac{2}{5} \times 100$

$= 40\ \%$

### Example 2

Convert 0.67 into percentage.

$0.67 = \frac{67}{100} \times 100$

$= 67\%$

### Percentage Increase and Decrease

A quantity can be expressed as a percentage of another by first writing it as a fraction of a given quantity.

### Example

A farmers harvested 250 bags of maize in a given season. If he sold 200 bags, what percentage of his crop does this represent?

### Solution

Let x be the percentage sold.

Then, $x/100 = \frac{200}{250}$

So, $x = \frac{200}{250} \times 100$

$= 80\%$

### Percentage Increase

A man earning Ksh. 4,800 per month is given a 25% pay rise. What will be the new salary?

Solution.

New salary $= \frac{25}{100} \times 4800 + 4800$

$= 1200 + 4800$

= Ksh. 6,000

### Percentage Decrease

A dress was costing Ksh. 1200 and now the same dress is costing Ksh. 960. What is the percentage decrease?

Solution

Decrease in cost = 1200 – 900 = Sh. 240

Percentage decrease $= \frac{240}{1200} \times 100$

$= 20\%$

## 2.6 References


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## 3.0 ESTIMATION, MEASUREMENT AND CALCULATION WITH ROUTINE METRIC MEASUREMENTS FOR WORK

### 3.1 Specific Learning Outcomes

At the end of the topic, the trainee should be able to:

i. Select and interpretation of measurement information in workplace tasks and texts
ii. Identify and select of routine measuring equipment
iii. Estimate and making measure using correct units
iv. Estimate and calculation using routine measurements
v. Perform conversions between routinely used metric units
vi. Use problem solving processes to undertake tasks
vii. Record information using mathematical language and symbols

### 3.2 Approximation

Consider a situation where numbers may not be given in exact figures. For example, in a place where people invited to a party is 250, the actual number could fluctuate between 240 and 260.

In this case, then, the approximate number is 250. Approximations involves rounding off and Truncations.

**Rounding Off**

**Example**
**Round off 395.184 to**

i) Nearest hundreds
ii) Four significant figure
iii) Nearest whole number
*iv) Two decimal place*

**Solution**

i) 400
ii) 395.2
iii) 395
iv) 395.18

**Truncation**

This is simply cutting off numbers to required decimal places or significant figures and ignoring the rest.

**Example 1**

Truncate 3.246 to 2dp

***Solution***

3.24 (Note that there is no rounding off)

**Example 2**

Truncate 561.7 to 2 Significant Figures

**Solution**

560 (note that zero is no counted)

**Estimation**

This is rounding off numbers so as to carry out calculations faster and get an approximate answer. The is meant to check the actual answer.

Example.
Estimate the answer to

$$\frac{152 \times 269}{32}$$

*Solution.*
*The answer should be close to:*

$$\frac{152 \times 269}{32} = 1,350$$

*The exact value is 1277.75 and if rounded off, it is 1300 which is a close estimate to the answer.*

**Accuracy and Errors**

In routine life, we encounter measurements such as:

i) Capacity of a water tank
ii) Length of a rope
iii) Mass of baby
iv) Room temperature
v) Time taken

Instruments for measuring include:

i) A calibrated container for measuring capacity
ii) A ruler (cm or m) for measuring Length
iii) A balance and weighing machine for Measuring Mass and weight
iv) Thermometer for measuring temperature.
v) Watch is used to measure time.

In using these instruments, there will be always a differences between the measured value and the actual value. Hence the measured value is always taken to be the approximates or the estimates of the actual value. The differences between the actual value and the measured value are called the **error.**

For accuracy, instruments such as micrometer screw gauge and Vernier calliper measure length more accurately than a ruler. Similarly, digital stop watch measures time to a higher accuracy than a wall clock. Again an electronic balance gives more accuracy of mass than ordinary balance.

### Absolute Error

When a measurement is state as 3.6cm, then it lies between 3.55 and 3.65cm
Then, the greatest error is 3.55 – 3.6 = -0.05
or 3.65 – 3.6 =+0.05
But we are going to ignore (±) sign simply because we are interested with only size of a possible error.

So, if we give the possible error without negative ) then this becomes an **absolute error.**

### Example

When a measurement is stated as 2.348 cm. then the measurement is to the nearest thousandths of a cm (0.001) and the absolute error will be as follows:

$\frac{1}{2}$x0.001=0.0005

Then, absolute error of a stated measurement is half of the least unit of measurement used.

### Length

This is the measurement between two point. The standard unit of measuring length is the Metre.

### Conversion of units of length

1 Kilometre (KM) = 1000metres
1 Hectometre (HM) = 100metre
1 Decametre (DM) = 10 metres
1 Decimetre (dm) = $\frac{1}{10}$metres
1 Centimetre (CM) = $\frac{1}{100}$ metres
1 Millimetre (MM) = $\frac{1}{1000}$ metres.

### Significant Figures

The accuracy considered while stating or writing a measurement may depend on its size. For example, the distance between town A and B may not be realistic to state as 153.27KM, in most cases we say 153KM. This is the distance expressed in 3 significant figures.

### Conversion of Unit of measurement to meters

Example
**Add and express the in metres 1.3km, 17m 12cm, 0.5Dm**

***Solution***
1.3km = 1.3 × 1000 = 1300m
17m 12cm = 12 cm =0.12 = 17 +0.12 =17.12m
0.5 Dm = 0.5 × 10 = 5m
1300m + 17.12m +5m = 1321.12 m

## 3.3 Perimeter

This is the total length of boundaries of a plane figure. It is expressed in the units of length.

### Perimeter of rectangle shapes.

If a rectangular has a length of *l* units and width of *b* units, then the formulae for its perimeter *p* is
$P = 2(l = b)$

### Perimeter of a square shapes

The perimeter can be expressed as (l × 4)

### Perimeter of triangular shapes

If the length of sides of a triangle is *a*, *b* and *c*, then the perimeter *p* is
$P = (a + b + c)$

### Perimeter of a circle

The formulae for finding the circumference of a circle is
C = where π = and *d* is the diameter of the circle.

### Example

Find the circumference of a circle whose radius is 7cm.
*Solution*

| | | |
|---|---|---|
| Circumference | = | πd |
| *d* | = | *2r* |
| | = | $\frac{22}{7}$ x2x7 |
| | **=** | **44 cm** |

### Length of an arc

The arc of circle is part of the circumference.

To find the length *l* of an arc of a circle which subtends an angle at the centre of the circle is given as

$l = \frac{\theta}{360} \times 2\pi r$

### Example

An arc of a circle subtends an angle of 60 at the centre of the circle. Find the length of the arc if its radius is 42cm. take ($\pi = 22/7$ )

### *Solution.*

$l = \theta/360 \times 2\pi r$ and $\theta = 60^0$

$= 60/360 \times 2 \times 22/7 \times 42$

$= 44$ cm

## 3.4 Rate

This is comparison of one quantity with another which is of a different kind. For instance, if a car takes 2hrs to cover a distance of 160 Kilometres, then the rate of the car is 80km/hr. Again if a bag of maize is Ksh. 2700, then the rate is Ksh. 30 per Kilo gram.

### Ratio
### Definition of Ratio

A ratio is a comparison between two (2) similar quantities. It is a numerical way of comparing quantities of the same kind. Ratios express quantities in the same units.
For instance, if two (2) quantities are in the ratio of three (3) to four (4) (generally written as 3:4); the first is  of the second. It is important to note that, before one quantity can be expressed as a fraction or a ratio of another, both quantities must be in the same unit(s).

The idea of ratios can be extended to cover more than two (2) quantity relationships. Thus, if three (3) weights are in the ratio 1:2:3, this means that, the second is twice the first and the third is three times the first. Additionally, to divide a quantity into parts in the ratio, say 2:4:7, then the first part should contain 2 units, the second 4 units and the third part 7 units; that is, the whole must be divided (2+4+7) = 13 units. The first part is then, the secondand the thirdof the whole.

This is a way of comparing two similar quantities. For instance, if Ali is 10 years old, and his brother Bashir is 14 years old, then Ali's age is 10/14 of Bashir's age and therefore we say their ages are in ratio of 10 to 14. Also written as 10:14.

### Example

i) Divide 1400 kg in the ratio 1:2:3.

### Solution:

The quantity should be divided into (1+2+3) =6 units

The **first** part = ⅙ of 1400 kg

= ⅙ × 1400 kg

= 233.33 kg

The **second** part = 2/6 of 1400kg

= 2/6 ×1400 kg

= 466.67 kg

The **third** part = 3/6 of 1400 kg

= × 1400 kg

= 700 kg

## 3.5 Proportion

This is comparison of two or more ratios. For example, if a, b and c are three numbers such that their ratio a: b:c = 2:3:5, then, a, b and c are said to be proportional. And their interpretation for relationship should be 2/a=3/b=c/5.
Example 1.
If a: b =3:4, and b:c = 5:7, work out a:c
a: b =3:4

$\frac{a}{3} = \frac{b}{4}$, $a = \frac{3b}{4}$

b:c = 5:7,

$\frac{b}{5} = \frac{c}{7}$

C =

a:b:c = $\frac{3b}{4} : b : \frac{7b}{5}$

(Multiply ratios by 20 to eliminate fractions)

= 15b : 20b : 28b (Divide the ratios by b)

a: b: c = 15 : 20 : 28

Therefore a: c = 15: 28

### Increase and decrease in a given ratio

This is where we give the ratio in fraction then multiply by the quantity.

### Example 1
Increase 20 in the ratio 5:4
The increased Quantity $= \frac{5}{4} \times 20$
$= 25$

### Example 2
Decrease the 45 in the ratio 7:9
The decreased Quantity $= \frac{7}{9} \times 45$
$= 35$

### Comparison of Ratios
For one to compare ratio, then the ratios are converted to fraction and then they are compared. Consider a:b = $\frac{a}{b}$

### Example
Which ratio is greater. 2:3 or 4:5

2:3 = $\frac{2}{3}$ and 4:5 = $\frac{4}{5}$
$\frac{2}{3} = \frac{10}{15}$ and $\frac{4}{5} = \frac{12}{15}$ then, is greater and so the ratio 4:5>2:3

### Distributing Quantities in a Given Ratio
This is dividing quantity in a given ratio. a:b:c. then a will have $\frac{a}{a+b+c}$ and b; $\frac{b}{a+b+c}$

and c will have $\frac{c}{a+b+c}$

### Example
72 hectares of land is divided among three sons in the ratio 2:3:4

### Solution
Total part will be 2+3+4=9
Then the first son will have $\frac{2}{9} \times 72 = 16$ha

$\frac{3}{9} \times 72 = 24$ha

$\frac{4}{9} \times 72 = 32$ha

### Direct and Inverse Proportion
### i) Direct Proportion

If two quantities are such that when one quantity increase/decreases, the other one also increases/ decreases.

### Example
If the cost of 1 cup is Ksh.100 then the cost of 3 such cups will be Ksh. 300
(if the number of cups increase, the cost increases also.)

### ii) Inverse proportion.

If two quantities are in such a manner that when one increases/decrease the other decrease/increase.

### Example
Consider dividing Ksh. 1200 among a number of students. If the number is 2 each student will take Ksh. 600. But if you are dividing among 3, each takes Ksh. 400 and for 4 each Ksh. 300.

(As the number of student increase, the amount each gets decreases)

## 3.6 References


- Bittinger, L. M., Beecher, J. A. & Johnson, B. L. (2019). Basic College Mathematics, 13th Edition. Pearson.
- Lial, M. L., Hornsby, J., McGinnis, T., Salzman, S. A. & Hestwood D. L. (2016). Basic Math, Introductory and Intermediate Algebra. Pearson.
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- Miller, J., O'Neill, M. & Hyde, N. (2015). Basic College Mathematics, 3rd Edition. McGraw Hill.

## 4.0 ROUTINE MAPS AND PLANS FOR WORK

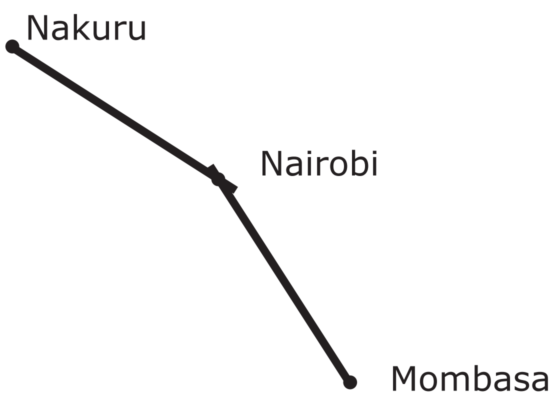
### 4.1 Specific Learning Outcomes

At the end of the topic, the trainee should be able to:

i. Identify of features in routine maps and plans
ii. Use Symbols and keys used in routine maps and plans
iii. Identify and interpret of orientation of map to North
iv. Demonstrate understanding of direction and location
v. Apply simple scale to estimate length of objects, or distance to location or object
vi. Give and receive directions using both formal and informal language

### 4.2 Scale Drawing, I Maps

Consider the illustration.




The distance between Nakuru and Nairobi is 142Km and from Nairobi to Mombasa is 450 Km. But if you can measure the distance in the illustration, you realize that certain length on the paper will represent a given distance on actual ground. Say, 1 cm rep 50km. and because 50km is equal to 5 000 000 cm, then, the statement is written in ratio 1: 5 000 000.

Therefore, the ratio of distance on a map to the actual distance on the ground is called scale of a map.

**The types of scales are:**

i) Statement scale e.g. 1cm rep 50Km
ii) Ratio scale e.g. 1: 5,000,000 also called Representative Fraction (R.F) = $^{1}/_{5,000,000}$
iii) Linear scale e.g. the one used in rulers (where there is division of a line into equal parts)

**Example 1**

The scale of map is give in a statement 1cm rep 4km. convert this to representative fraction.

**Solution**

1cm rep 4 ×1000cm

Therefore the ratio is 1: 4000 or in R.F = $^{1}/_{4000}$

**Example 2**

The scale of a map is given as 1:250,000. Write this as a statement scale.

**Solution**

1: 250,000 means 1cm on the map represent 250,000 cm on the actual ground.

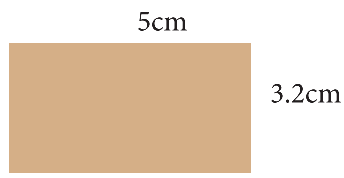
iv) Therefore, 1cm rep $^{250,000}/_{100,000}$ , i.e 1cm rep 2.5 km

### 4.3 Scale Diagrams

Consider the length of a classroom 10cm by 6.4cm. These measurements can be represented on a paper by scale drawing.

By considering a scale of 1cm rep 2m then the dimensions will be = 5cm by
6.4/2 = 3.2 cm

Figure 2




Suppose instead, you considered a scale of 1 cm rep 2.5 cm then the dimensions would be 10/2.5= 4 by 6.4/ 2.5= 2.56 (about 2.6)

Figure 3


Note: You may have observed that the bigger the scale the smaller the figure.

## 4.4 Bearing and Distance

captain while sailing and pilots while flying know direction by using magnetic compasses.

### Points of compass

The four main points are North (N), West (W), East (E) and South (S).
The angle between the main points of compass are $90^0$

### Compass bearing

When a direction a place from another is given in degree and in terms of the main points of compass, e.g., N $45^0$ E, then the direction is said to be given in compass bearing. The angles are measured clockwise from North or anticlockwise from South.

### True bearing

When measuring the degrees, the angles will be stated in three – figure bearing. E.g., N $45^0$ E is stated as $045^0$. This is the **True bearing.**

### Example

Kilo school is 12km from Sokomoko School on a bearing of $320^0$. Tiba dispensary is 10km from Kilo on a bearing of $120^0$. Find the compass bearing of Sokomoko from Tiba.

*Solution.*

## 4.5 References


- Bittinger, L. M., Beecher, J. A. & Johnson, B. L. (2019). Basic College Mathematics, 13th Edition. Pearson.
- Lial, M. L., Hornsby, J., McGinnis, T., Salzman, S. A. & Hestwood D. L. (2016). Basic Math, Introductory and Intermediate Algebra. Pearson.
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- Miller, J., O'Neill, M. & Hyde, N. (2015). Basic College Mathematics, 3rd Edition. McGraw Hill.

## 5.0 INTERPRETATION, DRAWING AND CONSTRUCTION OF 2D AND 3D SHAPES FOR WORK (MODEL MAKING)

### 5.1 Specific Learning Outcomes

At the end of the topic, the trainee should be able to:

i. Identify two dimensional shapes and routine three dimensional shapes in everyday objects and in different orientations
ii. Explain the use and application of shapes
iii. Use formal and informal mathematical language and symbols to describe and compare the features of two-dimensional shapes and routine three dimensional shapes
iv. Identify common angles
v. Estimate common angles in everyday objects
vi. Use formal and informal mathematical language to describe and compare common angles
vii. Use common geometric instruments to draw two dimensional shapes
viii. Construct routine three dimensional objects from given nets

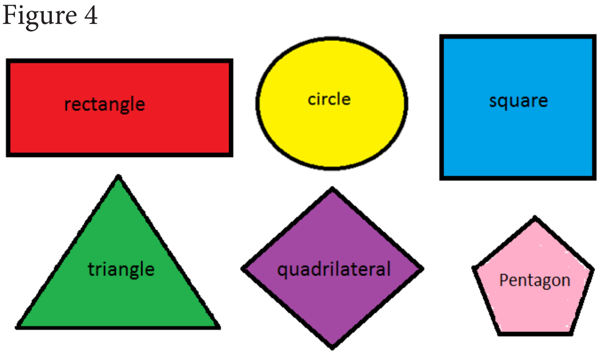
### 5.2 Dimensional Objects - 2D Shapes

In geometry, a shape or a figure with only length and width is a 2D shape. Their sides are only made of straight or curved lines. They are also called plane figures. Plane figures made of line are called **polygons**.

### 2D Shapes

Rectangle, circle, square, triangle, quadrilateral and pentagon are some examples of 2D shapes.

Figure 4



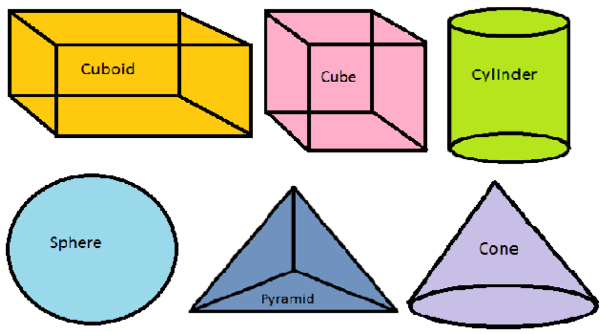

### 5.3 3D Shapes

One thing common in these objects is that they all have some length, breadth and height or depth. Thus they have three dimensions and so are known as 3D shapes. 3D shapes occupy space.

### Examples of 3D shapes

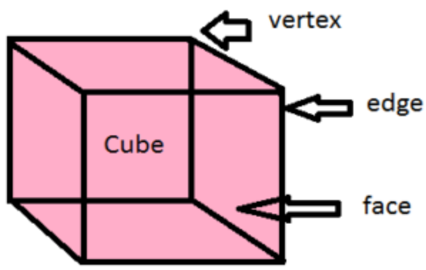
Cuboid, cube, cylinder, sphere, pyramid and cone are a few examples of 3D shapes

Figure 5




### Faces, Edges, and Vertices

The object below is a cube.
Figure 6

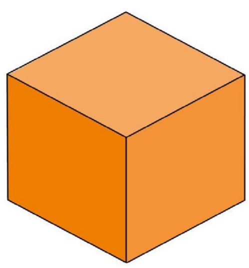



The corners of the cube are its vertices.
The 12 line segments that form the skeleton of the cube are its edges.

### Activity 1

You will need an object that assumes a shape of a cube such as a carton box.

Figure 7



a) How many faces does the carton have when open?
b) How many faces does the carton have when closed?
c) How many vertices does the carton have?
d) How many edges does the carton have?

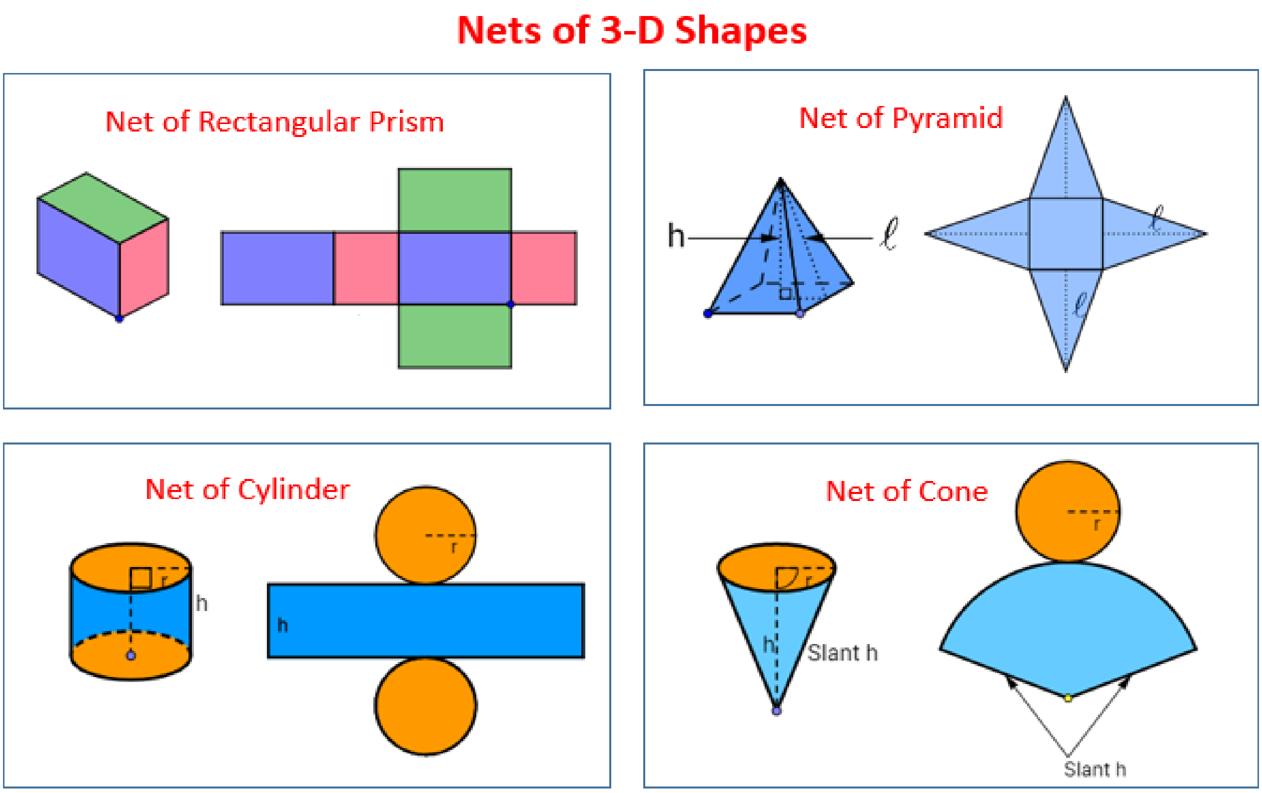
### Digital Corner

Using a digital device, use the link below to watch digital content on 3-D Objects.
https://www.youtube.com/watch?v=uVg-hvZhzUY

## 5.4 Nets for 3-Dimensional Shapes

A net is a two-dimensional pattern of a three-dimensional figure that can be folded to form the figure.

Figure 8




### Activity 2

**You require Manila Paper, Pair of Scissors, Marker pen and Cello tape/ Glue**

Trace the above Net of 3-D shapes illustrated above and use appropriate Linear Scale Factor to enlarge the nets on a Manila paper. Use scissors to cut the enlarged 3-D nets and model the 3-D objects. Count the corresponding faces, vertices and edges.

## 5.5 References


- Bittinger, L. M., Beecher, J. A. & Johnson, B. L. (2019). Basic College Mathematics, 13th Edition. Pearson.
- Lial, M. L., Hornsby, J., McGinnis, T., Salzman, S. A. & Hestwood D. L. (2016). Basic Math, Introductory and Intermediate Algebra. Pearson.
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- Miller, J., O'Neill, M. & Hyde, N. (2015). Basic College Mathematics, 3rd Edition. McGraw Hill.

## 6.0 INTERPRETATION OF ROUTINE TABLES, GRAPHS AND CHARTS

### 6.1 Specific Learning Outcomes

At the end of the topic, the trainee should be able to:

i. Identify routine tables, graphs and charts in predominately familiar texts and contexts
ii. Identify common types of graphs and their different uses
iii. Identify features of tables, graphs and charts
iv. Perform calculations to interpret information

### 6.2 Tables and Graphs

**Tables**

They contain the statistical data.

**Example**

The table 1 below shows the number of animals in a certain ranch.

**Table 1**

| Type of animals | Goats | Sheep | Cattle | Donkeys | Camels |
|---|---|---|---|---|---|
| Number of animals | 40 | 35 | 65 | 15 | 39 |

a. How many animals are there altogether? *194*
b. Which animals were the most? *Cattle*
c. Which animals were the least? Donkeys
d. How many more camels are there than sheep? *39 – 35 = 4 camels than sheep.*

**Graphs**

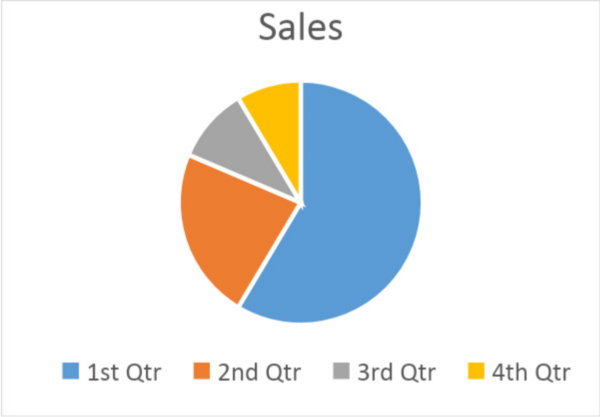
We use the information in the table and represents then on graphs.

The graphs include:

a. Circle graphs (pie charts)
b. Line graphs
c. Travel graphs (used to show relationship between distance travelled and time.
d. Bar graphs

Circle graphs.

Figure 9

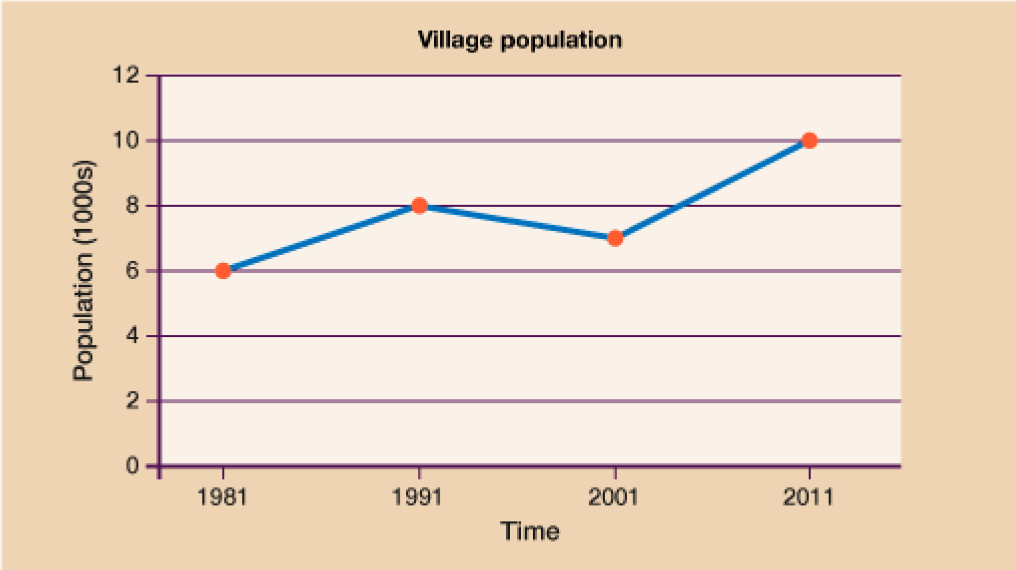


## 6.3 Line Graphs

### Interpreting Line Graphs

The graph shows information about the population of a village in thousands.

Figure 10




Use the graph to answer the question.

a. What was the population of the village in 1991? *8000*

b. What was the increase in population from 1981 to 2011? *10000 – 6,000 = 4,000.*

### Digital Corner

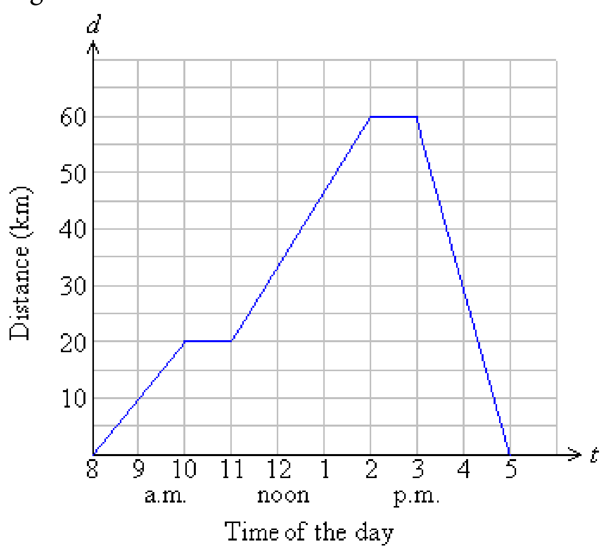
Use digital device to open the following link: [https://www.mathgames.com/skill/8.107-graph-a-line-from-a-function-table](https://www.mathgames.com/skill/8.107-graph-a-line-from-a-function-table). Plot digital lines graphs provided on the links. Trainee can select any other previously covered unit and interact with the content.

## 6.4 Travel Graphs

### Example

The following graph gives the distance of a cyclist from his home.

Figure 11




a) When did the cyclist leave home?
b) When did the cyclist return home?
c) How far away from home was he at 10 a.m.?
d) How far away from home was he at 11 a.m.?
e) How far away from home was he at 2 p.m.?
f) How far away from home was he at 3 p.m.?
g) At what times did he take a rest?

h) How far away from home was he at noon?
i) How far away from home was he at 5 p.m.?
j) Find his speed from:
   i) 8 a.m. to 10 a.m.
   ii) 11 a.m. to 2 p.m.
   iii) 3 p.m. to 5 p.m.
k) When was the cyclist travelling most quickly?

### *Solution*

a) The cyclist left home at 8 a.m.
b) The cyclist returned home at 5 p.m.
c) At 10 a.m., he was 20 km away from home.
d) At 11 a.m., he was 20 km away from home.
e) At 2 p.m., he was 60 km away from home.
f) At 3 p.m., he was 60 km away from home.
g) The cyclist took a rest between 10 a.m. and 11 a.m. and between 2 p.m. and 3 p.m.
h) At noon, he was about 33 km away from home.
i) At 5 p.m., the cyclist was at home. So, he was 0 km away from home.

$$\text{j. (i) Speed from 8 a.m. to 10 a.m.} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{20\text{ km}}{2\text{ h}} = 10\text{ km/h}$$

$$\text{j. (ii) Speed from 11 a.m. to 1 p.m.} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{40\text{ km}}{3\text{ h}} = 13\frac{1}{3}\text{ km/h}$$

$$\text{j. (iii) Speed from 3 p.m. to 5 p.m.} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{60\text{ km}}{2\text{ h}} = 30\text{ km/h}$$

k. The cyclist was travelling most quickly between 3 p.m. and 5 p.m.

## 6.5 Bar Graphs

### Example:

The following table 2 shows the number of visitors to a park for the months January to March.

Table 2

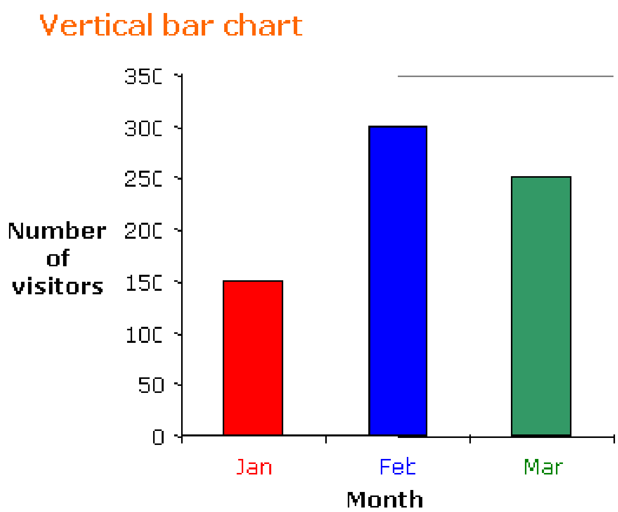
| Month | January | February | March |
|---|---|---|---|
| Number of visitors | 150 | 300 | 250 |

a) Construct a vertical and a horizontal bar chart for the table.
b) What is the percentage of increase of visitors to the park in March compared to January?
c) What percentage of visitors came in February compared with total number of visitors over the three months?

### Solution

a) If we choose a scale of 1:50 for the frequency then the vertical bar chart and horizontal bar chart will be as shown.

Figure 12




a) Increase in March compared to January is

x 100% = 66.67%

c) Percentage of visitors in February compared to the total number of visitors is

x 100% = 42.86%

$$\frac{300}{150+300+250} \times 100\% = 42.86\%$$

## 6.6 References


- Bittinger, L. M., Beecher, J. A. & Johnson, B. L. (2019). Basic College Mathematics, 13th Edition. Pearson.
- Lial, M. L., Hornsby, J., McGinnis, T., Salzman, S. A. & Hestwood D. L. (2016). Basic Math, Introductory and Intermediate Algebra. Pearson.
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## 7.0 COLLECTION OF DATA AND CONSTRUCTING ROUTINE TABLES AND GRAPHS FOR WORK

### 7.1 Specific Learning Outcomes

At the end of the topic, the trainee should be able to:

i. Determine data and variables to be collected
ii. Determine audience
iii. Select a method to collect data
iv. Collate information in a table
v. Determine suitable scale and axes
vi. Draft and draw graph to present information
vii. Check that data meets the expected results and context
viii. Report and discuss information using formal and informal mathematical language

### 7.2 Meaning of Statistical Data

Data is the foundation of all statistical works. Having the right, adequate and timely data enables researchers to undertake the necessary manipulations leading to effective decisions. Thus, data has to be defined.

Statistical data refer to those aspects of a problem situation that can be measured, quantified, counted, or classified. Statistical data are the basic raw material of statistics. Statistical data may relate to an activity of interest, a phenomenon, or a problem situation under study. Statistical data is obtainable as a result of the process of measuring, counting and/or observing phenomena.

Any object that is a subject of a given phenomenon, or activity that generates data through this process is termed as a variable. Hence, a variable is one that shows a degree of variability when successive measurements of a phenomenon are recorded.

### 7.3 Sources of Data

The sources of statistical data can either be through primary or secondary sources. These are explained as hereunder:

#### i) Secondary data

This type of data already exists in some form: published or unpublished - in an identifiable secondary source. The data had been previously collected for a different purpose other than the one at hand.

#### Advantages of secondary data

- It is obtainable quickly
- It is inexpensive compared to primary data sources
- Secondary data are usually available compared to primary data
- Secondary data usually enhance primary data
- Secondary data may assist in the achievement of the research objective(s)

#### Disadvantages of secondary data

- Sometimes, secondary data may not be in compatible units with the current assignment
- Its class definitions may not be usable
- At the same time, the available secondary data's measurement units may not match with the current assignment
- The available secondary data may be out-dated

As an attempt to address some of the limitations that would hinder the effective use of secondary data, it is important for the researcher to find answers to some of the following questions i.e.

- What was the purpose of the study?
- What information was collected?
- How was the information collected?
- Who collected the available information?
- How was the available information collected?
- How consistent is the available information with other information elsewhere?

#### ii) Primary data

This refers to those data which do not already exist in any form, and thus have to be collected for the first time from the primary source(s) or original source(s). By their very nature, these data require fresh and first-time collection covering the whole population or a sample drawn from it.

#### Advantages of primary data

- It is original in nature
- It is related to the research objectives at hand
- Provides an opportunity for the researcher to holistically the context in which data is being collected from

#### Disadvantages of primary data

- It is affected by time constraint

- It's an expensive process to obtain data from primary sources
- It is likely to be affected by poor response rate
- Sometimes, primary data may be unavailable

### Task 2: Explaining types of data

Types of data as an area try to categorize data into either quantitative or qualitative; depending on its nature. For either data, facts are normally considered as being of great importance.

### Quantitative data

Quantitative data refers to that data which can be quantified in definite units of measurement. These refer to characteristics whose successive measurements yield quantifiable observations. Quantitative data can further be categorized into two classes namely:

### i) Continuous data

This represents numerical values of a continuous variable. A continuous variable is the one that can assume any value between any two points on a line segment, thus representing an interval of values. The values are quite precise and close to each other, yet distinguishably different. For example, characteristics such as weight, length, height, thickness, velocity, temperature, tensile strength, among others, represent continuous variables.

### ii)Discrete data

A discrete variable is one whose outcomes are measured in fixed or definite numbers. Such data are essentially count data. These are derived from a process of counting, such as the number of items possessing or not possessing a certain characteristic, the number of customers visiting a departmental store on given days within a particular period of time, the incoming flights at an airport, and the defective items in a consignment received for sale, and so on, are all examples of discrete data.

### Qualitative data

A characteristic is qualitative in nature when its observations are defined and noted in terms of the presence or absence of a certain attribute in discrete numbers. Qualitative nature of given observations can either be nominal or rank data, i.e.

### i) Nominal data

These are the outcome of classification into two or more categories of items or units comprising a sample or a population according to some quality characteristic. Classification of students according to sex (males or females), of workers according to skill (skilled, semi-skilled, and/or unskilled), and of employees according to the level of education (Diploma holders, undergraduates, and post-graduates), etc, all result into nominal data.

Given any such basis of classification, it is always possible to assign each item to a particular class or a particular group and make a summation of items belonging to each class or group. The data so obtained through this classification or groupings are called nominal data.

### ii)Rank data

Rank data on the other hand refers to data that are the result of assigning ranks to specify order in terms of the integers 1, 2,3,4,5. ..., n. Ranks may be assigned according to the level of performance in a test, a contest, a competition, an interview, or a show based on certain observable characteristics. The candidates appearing in an interview, for example, may be assigned ranks using integers ranging from 1 to n, depending on their performance results arising from the interview. The ranks so assigned can be viewed as the continuous values of a variable involving performance as the quality characteristic.

## 7.4 Methods of Collecting Data

The choice of a suitable data collection method will largely depend on the type of data being targeted; either quantitative or qualitative. Whichever method of data collection to be used, researchers are always concerned about two (2) issues, that is:

*Theoretical issues* – these involve such matters as:

- The value of either type of data i.e. quantitative versus qualitative data
- The relative scientific rigor or manipulation required of the collected data etc

*Practical issues* – these are aspects that will affect either negatively or positively the actual data collection exercise. They include but not limited to:

- Credibility of findings, depending on the audience(s) targeted or the kind of stakeholders that one is working with
- Staff skills, especially the field staff, enumerators, supervisors
- Costs to be involved. And much of this

will depend on the amount of information needed, quality standards to be followed for the data collection, and the number of cases required for reliability and validity.
- Time constraints from the initial to the final stages of the research exercise. Whether one is interested in obtaining quantitative or qualitative data, a good data collection exercise requires considerable time to create and pretest questions and to obtain high response rates.

The principal methods available for data collection include:

### Survey method

Surveys are a very common method of data collection, especially where gathering information from large groups and the accompanying standardization is critical. The survey method is also referred to as paper-and-pencil instruments method. However, this is changing with the technological advances or emerging technologies.

Surveys can be undertaken through the use of questionnaires and interviews. Questionnaires can be self-administered, sent through post/mailed to the intended respondents for filling in the blanks/forms or filled through the use of technology e.g. telephone, computer-assisted etc.

The choice of an appropriate survey method would be influenced by the following factors:
- The type of respondent interaction required
- The complexity of questions to be used
- The resources available for use during the data collection exercise
- The project schedule or time available and so on.

### Advantages of surveys

- Surveys are easy to administer
- Can cover a wide range of topics or large samples
- Surveys are suited for gathering descriptive data as they are able to get 'beneath the surface'
- The use of surveys is relatively inexpensive
- Surveys could easily reveal sub-group differences
- The data obtained can be analyzed using a variety of platforms; including some of the emerging software

### Disadvantages of surveys

- In most cases, the data collected may only provide a general picture rather than an in-depth analysis
- Sometimes, self-reporting may lead to biased reporting of findings
- The results or findings may not provide adequate information on the context involved

### Experimentation method

In an experiment, a researcher assigns a treatment to a given case and then observes the response. In most instances, a control group (a group receiving no treatment or a placebo) may be used to compare the effectiveness of a treatment with the control group.

An experiment is the most systematic and popular method of data collection in the social sciences. This method allows for maximum control over extraneous variables or external interferences through the use of control and stimulus/treatment groups.

### Advantages of experimental study

- Experimental study findings are highly reliable
- The findings from an experimental study can be replicated elsewhere; thus proof to be highly consistent

### Disadvantages of experimental study

- In field experimentation, it is not possible to maintain the natural setting, thus having incomplete control of external variables
- The researcher must try to minimize disruption of normal behavior or occurrence of events
- There are considerable and immense potential ethical challenges while trying to ensure control in field experiments

### Observation method

An observation method involves the use of the human naked eye(s) or other technological devices/aids; in order to collect the intended original data. Consequently, they are methods by which an individual or individuals gather first hand data and/or information on programs, processes, or behaviors being studied.

Observation methods provide a holistic perspective of the case under consideration, i.e., an understanding of the context within which the subject operates. The behaviors of the subject or the contextual events are closely monitored. This may be especially important where it is not only the event that is of interest, but rather how that event may fit into, or may be affected by, a sequence of events.

### Advantages of observations

- Observations usually provide direct information about the behaviour of individuals and groups
- This method of data collection permits a researcher to enter into and understand a given situation/context
- As a method of data collection, it provides good opportunities for identifying unanticipated outcomes or occurrences
- Observations by their very nature exist in natural, unstructured, and flexible setting

### Disadvantages of observations

- Observation method is expensive and time consuming
- This method requires highly qualified, highly trained observers; especially content experts
- Observations may interfere or influence or affect the behavior of participants/subjects
- Selective perception of observer may distort the data being collected
- The behaviour or set of behaviours that would be observed may only represent a typical case

### Task 4: Describing sampling methods

Considering the problem at hand, a researcher will be faced with the option of examining all the subjects involved or a certain proportion of the whole. Either way, there are merits and demerits that would be realized from each; ranging from practicability to validity and reliability of the findings. The following part will enable us to understand this important stage that precedes the actual data collection.

### Meaning of sampling

Sampling is the process of selecting units desired for interrogation from a population of interest, so that by studying the sample, a researcher may fairly generalize the results back to the population from which they were chosen. This process could either be probabilistic or non-probabilistic based.

### Meaning of population

This can refer to all those people, households, items, subjects of interest; for researching purposes. Sometimes, all the items of interest are included in the study (i.e. undertaking a census) or take a selection (sample). Studying a census could be prohibitively time consuming, expensive and with limited enquiry. For these reasons, a census is of limited application for most businesses, economic or social studies; unless the population in question is small. Only in certain instances would a census be preferred over sampling; for example, if it is considered as a legal requirement, e.g. Kenya's census which is undertaken after every 10 years.

For purposes of developing a deeper understanding of the sampling process, there are other related terms to be included. That is:

*A parameter* is a characteristic of a population that you would wish to know. Most often this is a proportion, e.g. the proportion of people who might buy your product, the proportion of people that might vote in an election etc. sometimes; this can be any single numerical quantity.

For a population, it is the *mean* whereas for a sample, it is a *sample mean*.

**A** *Sample* refers to a small group of subjects/cases/items/persons selected from the population. The sample to be selected has to be representative and sufficiently large; for the results to be good enough for the intended purpose. The sample should be representative of the population, such that the results of the survey can be used to make generalizations about the population (i.e. making inference).

A sample measure is called a *statistic* whereas a population measure is called a parameter. For example, the average of a sample is often used to predict the mean of a population. And the proportion of a sample is used to predict the proportion of the population.

*A sampling frame* identifies a list of all members/items/cases of the population. This list might be conceptual, or might actually be many different lists.

### Reasons for sampling

i) Time constraint considerations
ii) Cost constraint considerations
iii) Practicability of the study using a census
iv) Achievement of a certain degree of accuracy of the anticipated findings

### Factors that influence sample representativeness

Sampling can either be based on probability or non-probability techniques. Sample size will be dependent on:

- The level of detail(s) required for analysis;
- The degree of accuracy of the findings expected; and
- The variability that is part and parcel of the population

The already mentioned two (2) approaches are explained below:

### Probability Sampling

This is a procedure that is designed in a manner such that, each person/item/subject is given a known and equal chance of inclusion in the resulting sample. Once that procedure is established, it is then followed for the selection of the persons/items/subjects. It involves such techniques as:

### Random sampling

In this method, each item in the population has the same probability (i.e. having some calculable chance) of being selected as part of the sample as any other items. For example, a researcher could randomly select 10 cases from the population of all possible valid cases within a range of 1 - 100 to use during interviewing. To be able to do this, a researcher could use a random number generator or simply put each number from 1 - 100 on a slip of paper in a hat or a basket, mix them up and finally draw out 10 cases. Random sampling can be done in two ways; i) with replacement and ii) without replacement. If it is done without replacement, an item is not returned to the population after it is selected and thus can only occur once in the sample. Otherwise, it might re-occur in the resulting sample.

To obtain a random sample, a list or sampling frame is required for guidance e.g. 1 – 100 in the example given above; thus achieving representativeness of the population.

### Advantages of random sampling

- Once the procedure is in place, there is no interference in the selection of the sample
- Representativeness of a sample is achieved.

### Disadvantages of random sampling

- As a sampling technique, it is only suitable for a relatively small population that is not geographically dispersed.
- It is affected by the problem of non-response; in that, not all persons/subjects/items/cases selected would be involved in the research.

### Systematic sampling

In this method, every $n^{th}$ element from the list or population is selected and included in the sample; starting with a sample element say $n$, which is selected randomly from the first $m$ elements. For example, if the population has 1000 elements and a sample size of 100 is required, then $m$ would be = 10. Then, if number 9 is selected randomly from the first 10 elements on the list, the sample would continue or be repeated down the list selecting every $9^{th}$ element from each group of 10 elements. A researcher is however cautioned that when using systematic sampling, s/he has to ensure that the original population list has not been ordered in a way that introduces any non-random factors into the sampling process.

As an illustration, a researcher might wish to obtain a suitable probabilistic sample for a particular study. The researcher would devise a criterion like every $14^{th}$ case be chosen out of the first 20 cases in a random list of all cases to be used for investigation purposes. The researcher would then keep adding 20 and select the $34^{th}$ case, the $54^{th}$ case, the $74^{th}$ case, the $94^{th}$ case and so on until the end of the list or population is reached.

### Advantages of systematic sampling

- Representativeness of the sample is realizable
- Every nth element has an equal chance of inclusion

### Disadvantage of systematic sampling

- It is a cumbersome process

### Stratified sampling

As a probability sampling technique, it is used when representatives from each sub-group or stratum or division within the population need to be represented in the sample. The first step in stratified sampling is to divide the population into sub-groups (strata i.e. plural for stratum) based on mutually exclusive criteria.

Random or systematic samples are then taken from each sub-group or division. The sampling fraction or portion for each sub-group or division may be taken in the same proportion as the sub-group or division has in the population. The final sample is therefore a collection of samples selected from each sub-group or division.

For example, if a person conducting a customer satisfaction survey selects customers at random from each customer type in proportion to the number of customers of that type in the population. This can further be illustrated as follows: If 40 samples are to be selected, and 10% of the customers are managers, 60% are users, 25% are operators and 5% are database administrators then 4 managers, 24 users, 10 operators and 2 administrators would be randomly selected. Stratified sampling can also sample an equal number of items from each sub-group or division. For example, a farmer randomly selected three fruit varieties out of each fruit collection in order to examine the extent of infestation by pests.

### Advantages of stratified sampling

- Accuracy of the results or responses is improved through careful stratified sampling
- Random sampling is possible

### Disadvantages of stratified sampling

- It is a cumbersome process
- In some instances, the sample may be unrepresentative depending on the selection guidelines

### Cluster sampling

This method of data collection is also called block sampling. In this method, the population that is being sampled is divided into groups/sub-groups called clusters. However, unlike in stratified sampling where these sub-groups are homogeneous (i.e. uniform) and based on a certain selected criterion, a cluster is as heterogeneous (i.e. lack uniformity) as possible to matching the population. Then probabilistically, a random sample is obtained from within one or more selected clusters.

For example, if an organization has 20 small projects currently being undertaken, an internal quality auditor looking for compliance to the ISO 9001:2015 Standard might use cluster sampling to randomly select 3 of those projects as representatives for the audit and then randomly sample particular aspects for auditing from just those 3 projects.

Another illustration would be based in a region like Nairobi County. If the behaviour of pupils learning within this county was to be studied, it would be argued that, pupils from Nairobi Primary School have many experiences in common with pupils from similar schools. This is would be a unique cluster of Nairobi County's pupils' population. One or more of these clusters are selected randomly and a sample selected (i.e. either through random or systematic technique). If circumstances permit, a census would be carried within the selected clusters.

Cluster sampling can tell us a lot about that particular cluster. However, unless the clusters are selected randomly and a lot of clusters sampled, generalizations cannot always be made about the entire population. Otherwise, there would be biases when selecting the sample that would not allow for statistically valid generalizations.

### Advantages of cluster sampling

- It is a convenient technique
- It is cost effective

### Disadvantages of cluster sampling

- This technique mostly suffers from sample non-representativeness
- Population generalizations may be impossible

### Multi-stage sampling

Multistage sampling refers to a sampling plan where sampling is carried out in stages using smaller and smaller sampling units at each stage. In a two-stage sampling design, a sample of primary units is selected and then a sample of secondary units is selected within each primary unit. In order to achieve a representative sample, further stages could be added or further stratification (i.e. based on other characteristics) could be used at some or all of the stages. In a country like Kenya for example, it would be possible to have partitions based on national, county, constituency, ward etc levels. An important consideration in this technique is how partitioning takes place at each stage.

Multistage samples are used primarily for cost or feasibility (practicality) reasons.

### Advantages of multi-stage sampling

- As a sampling technique, sample representativeness is achieved
- The practicability of the data collection exercise can be determined beforehand

### Disadvantages of multi-stage sampling

- Sometimes, the objectives of the research might be unrealized as the criteria being used for sampling would put off possible participants

- The technique may proof to be difficult to undertake
- It is normally affected by cost and time constraints

### Non-Probability Sampling

Unlike probability sampling where a predetermined procedure is used, non-probability sampling relies heavily on an individual's conscious decision to include or exclude certain persons/items/subjects/cases.

It is however important to note that, a well-conducted non-random sampling technique can produce acceptable results more quickly, and at a lower cost, than a probability sample. Some of the techniques involved include:

### Quota sampling

Under this method, interviewers are sent into an area with instructions that require them to interview a certain number of people who meet particular characteristics. These characteristics of the population for instance age, gender, occupation, tribe, among other, are considered important for obtaining a quota. Based on the characteristics, a proportion of each in the population can then be obtained from secondary sources. Having achieved a certain number or quota from the population, an interviewer is left with the responsibility of selecting the final list for use.

For example, they might be needed to choose 9 people; 5 men and 4 women. In addition, the number should be composed of 4 working and 5 non-working nationals etc. The underlying idea is that, if the quotas accurately or to a larger extent reflect the population, then the sample will be considered representative.

### Advantages of quota sampling

- It is easier and quicker to obtain quota samples i.e. it is cost- and time-effective
- It is easy to supervise the data collection exercise

### Disadvantages of quota sampling

- Information on population characteristics obtainable from secondary sources might be unreliable.
- If the desired population characteristics are not readily identifiable, it will proof to be costly and time wasting

### Judgmental sampling

It is also called purposive sampling. Under this method, a researcher or a person interested in obtaining a sample uses his/her knowledge or experience to select the items to be included in the sample and ultimately sampled. Thus, there is no element of chance and judgment is used to select participants. For example, a teacher may identify certain students to represent his/her class for a certain purpose.

To illustrate this, based on experience, an internal quality auditor may know which areas are more prone to have non-conformances or which areas have had problems in the past or which areas are at a higher risk to the organization.

### Advantages of judgmental sampling

- It is an easier and quicker technique of obtaining the required samples i.e. it is cost- and time-effective

### Disadvantages of judgmental sampling

- It may be difficult to use its findings for generalization purposes
- Assumptions made might not always hold

### Snowball sampling

This technique as the name suggests, is undertaken through stages; moves on from an initial starting point (snowballs) to identify desirable participants for investigation. Certain circumstances necessitate the use of this technique; like where desired participants are difficult to identify and are often rare or difficult to locate.

Suppose we want to trace the roots of terrorism attacks in Kenya. The initial attempt would be to get individuals who seem to fit the given description or individuals that have the right contacts. Once the starting point has been established, further interrogations are made to narrow down the eligibility list and lead to other contacts.

### Advantages of snowball sampling

- When leads are precise, it becomes a fast and easy technique of capturing data
- It is possible to unearth almost forgotten phenomena

### Disadvantages of snowball sampling

- It may proof difficult if the subjects do not fully fit into the description required
- It is hindered by non-response

### Convenience sampling

Under this sampling technique, a sample is obtained on the basis that it is easy and convenient for study purposes. For example, if a researcher wants to get the opinion quickly before doing an in-depth investigation, then again select a few individuals that are 'convenient' to work with, it will finally proof to be a helpful approach. This approach would achieve good results with a limited sample. However, generalizing those findings might proof difficult.

### Advantages of convenience sampling

- It is quicker to assess the direction that a study might take
- It is an easier and quicker technique of obtaining the required information i.e. it is cost- and time-effective

### Disadvantages of convenience sampling

- It suffers from non-representativeness of samples used
- It is impossible to generalize findings

### Explaining the meaning of classification of data

The raw data, that has been collected from varied sources and put or recorded randomly, does not give a clear picture of the whole. In order to locate similarities and reduce mental difficulties that are likely to be experienced, it is only wise to classify that data. Through classification, the data is condensed by leaving out unnecessary details. Classification will facilitate comparison between different sets of data; by clearly showing the different points of agreement and disagreement.

### Definition of data classification

Data classification is the process of arranging things in groups or classes based on certain characteristics, like to their resemblances; thus giving expression as to the unity of attributes that may exist amongst a diversity of individuals / subjects / observations.

Data can be classified according to sex (males/ females), marital status (married/unmarried), place of residence (rural/urban), Age (0–5 years, 6–10 years, 11–15 years, etc.), profession (agriculture, production, commerce, transport, doctor among others); depending on whether it is qualitative or quantitative.

### Guidelines for data classification

i) The definition of classes should precise and free of any likely doubts. This will eliminate all doubts while including a particular item in a class.

ii) All the classes chosen should preferably have equal width (i.e. class intervals). It is only in a few special cases, where classes of unequal width are used.

iii) The class-limits (integral or fractional) should be selected in such a way that no single value of the item in the raw data coincides with the value of the determined class-limits.

iv) The number of classes to be used for classification should preferably be between 10 and 20, i.e., neither too large nor too small.

v) The classes should be accommodative of each value of the raw data; without leaving out extreme values

vi) The classes should be mutually exclusive and non-overlapping, that is each item of the raw data should fit only in one class.

vii) The classification mode to be adopted must be suitable for the object of inquiry.

viii) The classification should be flexible and items included in each class must be homogeneous i.e. uniform in nature.

ix) Width of class-interval is determined by first fixing the number of class-intervals and then dividing the total range by the number of classes.

### Task 2: Explaining the construction of a frequency distribution table

A frequency distribution table is an important tool for classifying given data. Such a table offers a good picture of the nature of the data distribution in question; whether grouped or ungrouped.

## 7.5 Frequency Distribution Table

A frequency distribution table is a tabular presentation of given data; showing the particular data items or observations, either individually or in groups; together with their accompanying frequencies (i.e. the number of times that they appear in the data set).

The process of constructing a frequency distribution table involves a number of steps. In the case of ungrouped data (and especially involving limited data), the process is easier in that, we only identify the number of times that an item or observation occurs.

For example, consider this data distribution.
10, 3, 6, 7, 3,3, 11, 5, 4, 5,5,5,4,4,4,4, 8,9,10,10,10,10, 10,10, 11,3,4,6,5,7

This distribution has 30 data items. This data can first be arranged in an ascending or descending order. Otherwise if the data is limited and clear to the user, this step would be ignored. Note the individual numbers and ascertain their corresponding frequencies. Thus,

Table 4

| **Number** | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|
| **Frequency** | 4 | 6 | 5 | 2 | 2 | 1 | 1 | 7 | 2 |

To test the correctness of the frequency items, count the number of data items and compare that with the values appearing on the frequency distribution table. The total number of items must be equal to the total frequency.

For bulk data, it would be appropriate to consider an appropriate classification format; especially if it can be grouped into categories or classes. For a case of this nature, a number of steps would be followed to properly classify the data under examination. These steps are as outlined below:

1) Decide on the number of classes your frequency table will contain.
2) Determine the class width by dividing the range by the number of classes.
3) Locate the starting point by selecting it as the lower limit of the first class; either the lowest score or a convenient value slightly less than the lowest score.
4) Add the class width to the starting point to get the second lower class limit.
5) List the lower class limits in a vertical column, along with the upper class limits.
6) Complete the table by counting up class-interval frequencies and filling them in the table

For example, consider the data provided below and construct a frequency distribution table for it.

15,17,20,33,45,28,44,16,27,17,15,23,21,37,39,42,38, 39,24,18,25,35,40,19,25,36, 43, 16,45,42.

*Solution:*

There are 30 data items. First arrange the data set in ascending order of magnitude. This makes it easier to determine the range of the data distribution given.

15,15,16,16,17,17,18,19,20,21,23,24,25,25,27,28,33, 35,36,37,38,39,39,40,42,42,43,44,45,45

For this distribution, we intend to have 6 classes. Having the range as 30, the desirable class interval shall be 30/6 = 5. Then follow the above listed steps. We will finally get a frequency distribution table 5 as the one shown below:

Table 5

| **Number** | 15 -20 | 21 -26 | 27 -32 | 33 -38 | 39 -44 | 45 -50 |
|---|---|---|---|---|---|---|
| **Frequency** | 9 | 5 | 2 | 5 | 7 | 2 |

## 7.6 Methods of Tabulating Data

Tabulating is a way of processing information or data by putting it in a table. The tabulation process will require a table, or chart, with rows and columns based on certain characteristics. It expresses the data in concise and attractive format which can be easily understood and used for comparison purposes.

Fundamentally, the central objectives of tabulation include but not limited to:

i) To enable the researcher, carry out investigation for a particular data set
ii) For comparison purposes
iii) To locate omissions and errors in the dataset if any
iv) To use space economically
v) To study the trend being shown by the data set
vi) To simplify data for easier of further treatment or manipulation
vii) To use data future reference purposes

Whenever data is provided, its close examination will determine whether it would be presented as grouped or ungrouped. As such, the two (2) approaches shall be considered for tabulation purposes. Hence, the data can be classified into:

### Grouped data

The way of tabulating a pool of data of a variable and their respective frequencies side by side is called a frequency distribution of the given data set.  A frequency distribution is thus "a statistical table which shows the sets of all distinct values of the variable arranged in order of magnitude, either individually or in groups, with their corresponding frequencies side by side".

The data set in question can be condensed by putting it into smaller groups, or, classes. The smaller groups or classes could be characterized by a range of data. The ranges being shown by the groups or classes are appropriately referred to as *class-intervals*. The number of items which fall into any class-interval is called its *class frequency*.

The number of classes for a particular data set is determined by the expression, that is;

Number of classes $= \frac{Range}{Class\ Size}$

The maximum and minimum values of a class-interval are called the *upper class limit* and the *lower class-limits* respectively. Class boundaries are the true-limits or exact limits of a class interval. This concept is greatly associated with grouped frequency distribution, where there is normally a gap between the upper class-limit and the lower-class-limit of the next class or the lower class-limit and the upper class-limit of the preceding class.
This can be determined by using an expression as shown below:

Lower class boundary = lower class-limit $- \frac{1}{2d}$

Upper class boundary = upper class-limit $+ \frac{1}{2d}$

Where $d$ = common difference between the upper class-limit of a class-interval and the lower class limit of the next higher class interval.
For example, consider a raw data set having class intervals; 30 - 34, 35 – 39 and so on.

The true class boundaries would be:
First determine the value of $d$ = 35 -34 = 1
Then, the lower class boundary =lower class-limit –

Substituting the values in the expression
The lower class boundary = 30 – ½ (1)
= 30 – 0.5
= 29.5

Upper class boundary = 34 + ½ (1)
= 34 + 0.5
= 34.5

The tabulation of raw data by dividing the whole range of observations into a number of classes and indicating the corresponding class-frequencies against the class-intervals is called a grouped frequency distribution. When this is done using a table, then the resulting diagram is regarded as a grouped frequency distribution table.

Below is an example of a grouped frequency distribution table 6:
Table 6

| **Marks** | 0 - 10 | 10 - 30 | 30 - 50 | 50 - 80 | 80 - 90 | 90 - 100 |
|---|---|---|---|---|---|---|
| **No. of Students** | 4 | 12 | 20 | 8 | 4 | 2 |

## Important considerations for grouping data

1) Consider class-intervals of this nature; 20-25, 25-30; 50 - 100, 100 - 150; 30 - 40, 40 – 50 and so on. These are all upper limit exclusive type of class boundaries i.e. it only includes the upper limit values and not the lower limit values. That is, an item exactly equal to 25, 100 and 40 are put in the intervals 25 - 30, 100 - 150 and - 50, respectively and not in the class intervals 20 - 25, 50 - 100 and 30 - 40, respectively. Similarly, 25 is included and 20 excluded (lower limit) in "above 20 but not more than 25" class-interval. In the exclusive type, the class-limits are continuous, i.e., the upper-limit of one class-interval is the lower limit of the next class-interval and class limits of a class-interval coincide with the class boundaries of that class-interval. It is suitable for continuous variable data and facilitates mathematical computations.

2) Given classes like 50 - 59, 60 - 69, 70 - 79, etc., are all of the inclusive type. Here, both the upper and lower class-limits are included in the class-intervals, for instance 50 and 59 both are included in the class-interval 50 - 59. This is suitable for discrete variable data. There is no problem as to which class an item belongs but the idea of continuity is lost. To make it continuous, the exact class boundaries have to be determined and it can then be expressed as (49.5 - 59.5), (59.5 - 69.5), (69.5 - 79.5) and so on.

3) For an 'open-ended' class-interval, either the lower limit of the first class-interval or, upper limit of the last class-interval, or, both is missing. It is difficult to determine the mid-values of the

first and the last class-intervals without a smart assumption. If the other closed-ended (i.e. those in the middle) class-intervals have equal width, then we can assume that even the open-ended class-intervals (on either end) also have the same common width as that of the closed class-intervals. Grouped frequency distributions are kept open-ended when there is limited number of items scattered over a long interval.

4) Sometimes, unequal class-intervals are preferred only when there is a great fluctuation in the data distribution. For example, data set having class-interval like; (0 – 3), (4 -5), (6 – 8) and so on.

**Ungrouped data**

Ungrouped data considers the individual items of a given data set. To begin with, the data is ordered in either ascending or descending format. Having either of the formats, one is able to identify the number of times that particular data items appear. This will form the basis for determining the frequency of the individual data items in the distribution. Tally marks mostly used for indicating the number of times that individual observations occur.

A tally mark is an upward slanting stroke (/) which is put against a value each time it occurs in the raw data set. The fifth (i.e. 5th) occurrence of the value is represented by a cross tally mark (\) as shown across the first four tally marks. The tally marks are counted and the total of the tally marks against each value or data item, is its frequency.

**Example:**

Arrange the data in ascending order.
9,4,5,1,8,3,4,7,5,9,6,3,8, 3,8,6, 9, 1,3,4,2,6,5,2,1,4,4,2, 3,1,4,5,6,5,9,8,7,3,8,6.

**Solution**

1,1,1,1,2,2,2,3,3,3,3,3,3,4,4,4,4,4,4,5,5,5,5,5,6,6,6,6,6, 7,7,8,8,8,8,8,9,9,9,9.

In total, there are 40 data items. The individual data items appear a different number of times. The number of times of occurrence of the items represents their frequencies.

Using tally marks, this information can be presented as shown below:

Figure 7: Tally Marks

| Numbers | Tally | Frequency (f) |
|---|---|---|
| 1 | //// | 4 |
| 2 | /// | 3 |
| 3 | ~~////~~ / | 6 |
| 4 | ~~////~~ / | 6 |
| 5 | ///// | 5 |
| 6 | ///// | 5 |
| 7 | // | 2 |
| 8 | ///// | 5 |
| 9 | //// | 4 |

Alternatively, the frequency distribution table shown below can be used in order to use the available space economically.

Table 8

| **Numbers** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| **Frequency** | 4 | 3 | 6 | 6 | 5 | 5 | 2 | 5 | 4 |

## 7.7 Constructing Routine Tables and Graphs for Work

Once data has been collected, it can be presented as text, in tables, or pictorially using graphs and charts. That is,

i) Textual presentation (like in an accident scene; 24 fatalities and 3 injuries, 20 per cent of other unreported cases etc)
ii) Tabular presentation using tables or charts
iii) Graphical presentation (quantitative data may also be presented graphically by using bar charts, pie diagrams, pictographs, line diagrams and so on)

Tables are usually the best way of showing structured numeric information, whereas graphs and charts are better for showing relationships, making comparisons and indicating trends. Even in instances where a graph or chart is used, it is usual to include a table to show the origin of the data.

## Charts

A chart usually gives an idea of the value(s) as well as a visual indication of how the value(s) are changing. Effective and correct use of charts requires an understanding of the following:

1) Type of data to be presented
2) The key feature(s) to be brought out or portrayed
3) How the information would be used
4) Who the targeted audience would be

## Pictograms / pictographs

This is a pictorial data presentation, where symbols or pictures or sketches of the things being described, represent the data set items; as opposed to the use of bars or pie. E.g. the use of such shapes as Ơ (to represent fruits),

For example, present the information given using a pictogram. (Hint: Ơ represents 2 fruits and ơ represents 1 fruit)

We have a fruits' distribution as: 3, 6, 8, 11, 15, 12 and 19

*Solution:*

The available number of fruits can be presented thus:

Figure 13: Pictograms / Pictographs

| Key | Number / frequency |
|---|---|
| Oranges | Ơ ơ |
| Avocadoes | Ơ ƠƠ |
| Apples | Ơ ƠƠƠ |
| Passion | Ơ ƠƠƠƠơ |
| Lemons | Ơ ƠƠƠƠƠƠơ |
| Mangoes | Ơ ƠƠƠƠƠ |
| Pineapples | Ơ ƠƠƠƠƠƠƠƠơ |

## Advantages of pictograms

- Immediate visual perception is achieved i.e. they are eye-catching
- The use of picture or visual presentation of data can simply tell a story
- Visual presentation of data is universally accepted without knowledge of a particular language.

## Disadvantages of pictograms

- They are not very accurate and require interpretation
- Pictograms are only suitable for presenting limited amount of data
- Pictograms do not cater for the different categories of data items; for example, male versus female

## Pie charts

A Pie chart is a visual tool that is used for showing proportions arising from a given data distribution or series. For example, percentages of pupils giving different responses when evaluating a course, the composition of domestic animals in a certain home etc. They are used to show the proportions of a whole. Also, they are best used when there are only a handful of categories to display.

A circle (i.e. the pie) is divided into sectors or segments; one sector/segment for each category of the data given. The size of each sector/segment is determined by the frequency (i.e. the number of observations) of the category and measured by the angle that is representative of the sector/segment. As the total number of degrees in a circle are 360°, the angle given to a segment is 360◦ times the fraction of the data in the category, that is,

$$\text{Angle} = \frac{\text{Number of items in a particular category}}{\text{Total number of items in a sample } (n)} \times 360°$$

## Example:

The data given below shows the choice of means of travel (i.e. mode) to the workplace by different people. Present the data using a pie chart.

Table 10

| Mode | Frequency (f) |
|---|---|
| Car | 11 |
| Walk | 8 |
| Bike | 4 |
| Bus | 6 |
| Metro | 4 |
| Train | 2 |
| **Total** | **35** |

*Solution:*

First, determine the angle for representing each mode of travel:

Using car; $\frac{11}{35} \times 360° = 113.14° \cong 113°$

Walking, $\frac{8}{35} \times 360° = 82.29° \cong 82°$

Using bike, $\frac{4}{35} \times 360° = 41.14° \cong 41°$

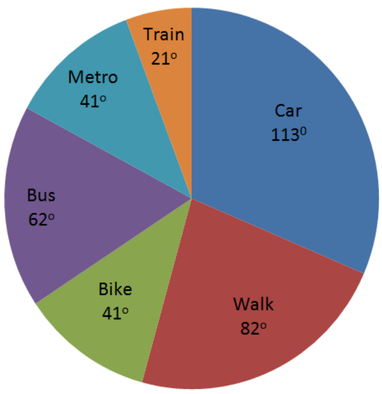
Using bus, $\frac{6}{35} \times 360° = 61.71° \cong 62°$

Using metro, $\frac{4}{35} \times 360° = 41.14° \cong 41°$

Using train, $\frac{2}{35} \times 360° = 20.57° \cong 21°$

After ascertaining the angle in degrees representing each mode of travel; then draw the pie chart using the calculated angle segments for the various modes of travel.

### Figure 14: Pie Chart




### Advantages of pie charts

- It is simple to illustrate
- It is easy to understand

### Disadvantages of pie charts

- Pie charts only show relatively limited amount of data
- Visually, one cannot accurately evaluate the relative sizes of the pieces

### Bar Charts

A Bar chart provides a clear visual display of simple results. Bar charts are normally used when the horizontal axis is composed of categories of information. For example, male or female; students attending study support sessions and those who don't; ethnic groups, individual pupils, people's occupations etc.

A stacked bar chart can be used if some sort of improvement in a category needs to be displayed. Remember that, if the bars are not separated by spaces, the chart is referred to as a *histogram*, rather than a bar chart.

A bar chart can then be drawn by following the steps given below:

i) First, decide on what information is to be presented on each axis of the chart. By convention, the variable being measured is put on the horizontal ($x$–axis) whereas the frequency is reflected on the vertical ($y$–axis).

ii) A decision is the made regarding the numeric scale for the frequency axis. This axis represents the frequency in each category, shown by its height. The frequency scale therefore has to start at zero and include the largest frequency. It is common to extend the axis values slightly above the largest value of the frequency. This is necessary so that, one does not draw to the edge of the graph.

iii) Having decided on a range for the frequency axis (i.e. from zero to the highest frequency), then we need to decide on a suitable number scale to label this axis (i.e. the vertical axis). This should have sensible values, for example, 0, 1, 2. . . or 0, 10, 20 . . . , or other such values that would make sense for the given data.

iv) Draw the axes (i.e. vertical and horizontal axes) and label them appropriately.

v) Draw a bar for each category of data.

When drawing the bars, it is significant to ensure that:

- The width of each bar is the same consistently, and
- The bars are separated from each other by equally sized gaps

### Example:

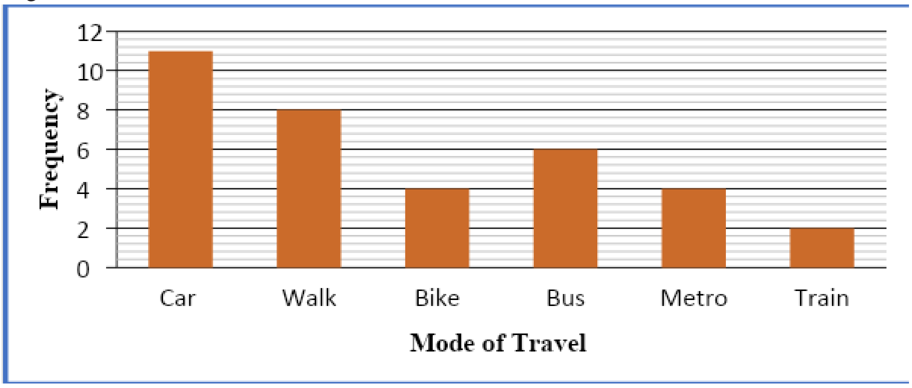
Using the information from the previous example, draw a bar chart:

Table 11

| Mode | Frequency *f* |
|---|---|
| Car | 11 |
| Walk | 8 |
| Bike | 4 |
| Bus | 6 |
| Metro | 4 |
| Train | 2 |
| **Total** | **35** |

**Solution**

Figure 15: Bar Chart




### Advantages of bar charts

- Are simple to construct
- They used for investigating specific comparison
- Bar charts compare categorical data
- Give a clear visual impression regarding the data distribution or series

### Disadvantages of bar charts

They cannot be used to present continuous data

### Histogram

A histogram is applicable for the presentation of a continuous data variable unlike the case of bar charts. A histogram is different from a bar chart in two instances:

- The horizontal (x-axis) is a continuous scale. As a result of this, there are no gaps between the bars (unless there are no observations within a class interval);
- The height of the rectangle is only proportional to the frequency if the class intervals are all equal. In this sense, while using histograms, it should be remembered that, it is the area of the rectangle that is proportional to their corresponding frequency.

In order to present data using a histogram, first produce a frequency table which collects all the data together in an ordered format. Thereafter, follow these steps:

i) Find the maximum frequency and draw the vertical ($y$–axis) starting from zero to this maximum frequency value, including a sensible numeric scale.

ii) The range of the horizontal ($x$–axis) should include not only the full range of observations but also the full range of the class intervals from the derived frequency table.

iii) Draw a bar for each group in your frequency table. Remember that, it is important to have bars with the same width and touch each other (unless there are no data in particular classes).

### Example:

Present the data given in a frequency distribution table below, using a histogram

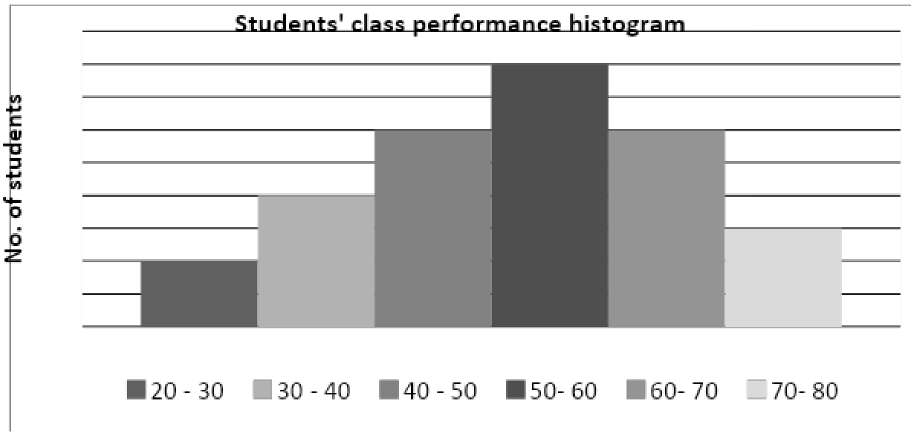
Table 12

| Marks | Frequency (f) |
|---|---|
| 20 - 30 | 4 |
| 30 - 40 | 8 |
| 40 - 50 | 12 |
| 50- 60 | 16 |
| 60- 70 | 12 |
| 70- 80 | 6 |
| 80- 90 | 2 |
| **Total** | **= 60** |

### Solution

Follow the steps provided above and draw a histogram for the given frequency distribution.

Figure 16: Histogram




## Advantages of a histogram

- It provides a clear visual representation of the data distribution given
- From a histogram, it is easy to spot the modal or most popular class in the data, i.e. the one with the highest peak or highest frequency
- It is also easy to spot simple patterns in the data distribution
- Histograms also allow us to make early judgments as to whether all our data come from the same population or different populations.

## Disadvantages of a histogram

- It is not always easy to present open-ended data using a histogram
- Histogram shapes are inconsistent as they largely depend on the definition of the classes

## Graphs

Graphs include the following:

### Line graphs

A line graph is an improvement from the bar chart. Line graphs are appropriate when the horizontal axis is continuous rather than being separated into categories. A line graph could be used to show progress over time. For example, the development of a measured skill each week over a ten-week period, the increase in height of a seedling over a number of weeks and so on.

**Example:**

Present the data given in table 13 below, using a line graph.

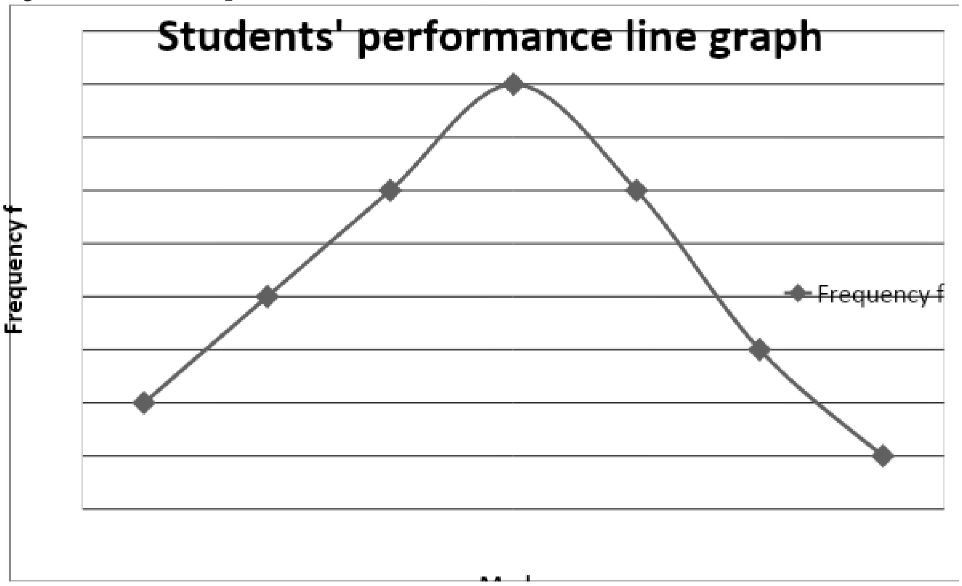
Table 13: Frequency Distribution Table

| Marks | Frequency (f) |
|---|---|
| 20 - 30 | 4 |
| 30 - 40 | 8 |
| 40 - 50 | 12 |
| 50- 60 | 16 |
| 60- 70 | 12 |
| 70- 80 | 6 |
| 80- 90 | 2 |
| **Total** | $\Sigma f = 60$ |

***Solution*:**

The steps to follow are similar to those of drawing a histogram. Thus,

Figure 17: Line Graph




### Advantages of line charts/graphs

- They analyze trends of data
- They show patterns of data as well as exceptions

### Disadvantages of line graphs/charts

- They are difficult to interpret
- discontinuous changes cannot easily be monitored

### Frequency polygon

A frequency polygon is simply a natural extension of a histogram. The only difference between the two is that, rather than drawing bars, each class is represented by one point (i.e. the mid-points of the classes) and these are joined together by straight lines.

Alternatively, the tops of the mid-points of the bars in a histogram are joined with straight lines; resulting to a frequency polygon.

The procedure for drawing a frequency polygon is similar to that for producing a histogram i.e.

i) First complete the frequency distribution table
ii) Determine the mid-points for the given class intervals
iii) Plot the axes

- The x-axis (horizontal axis) should contain the full range of the classes from the frequency distribution table
- The y-axis (vertical axis) should contain the range from 0 to the maximum frequency

iv) Plot points: pick the mid-point of the class interval on the x-axis and go up until you reach the appropriate frequency value on the y-axis and mark the point. Repeat this process for each class.
v) Join adjacent points together using straight lines

Alternatively, one can obtain a frequency polygon from a histogram. This is done by joining the mid–points of the top of each bar with a straight line. As such, a frequency polygon can be drawn on top of a histogram.

### Example:

Present the data given in a frequency distribution table 14 below, using a frequency polygon.

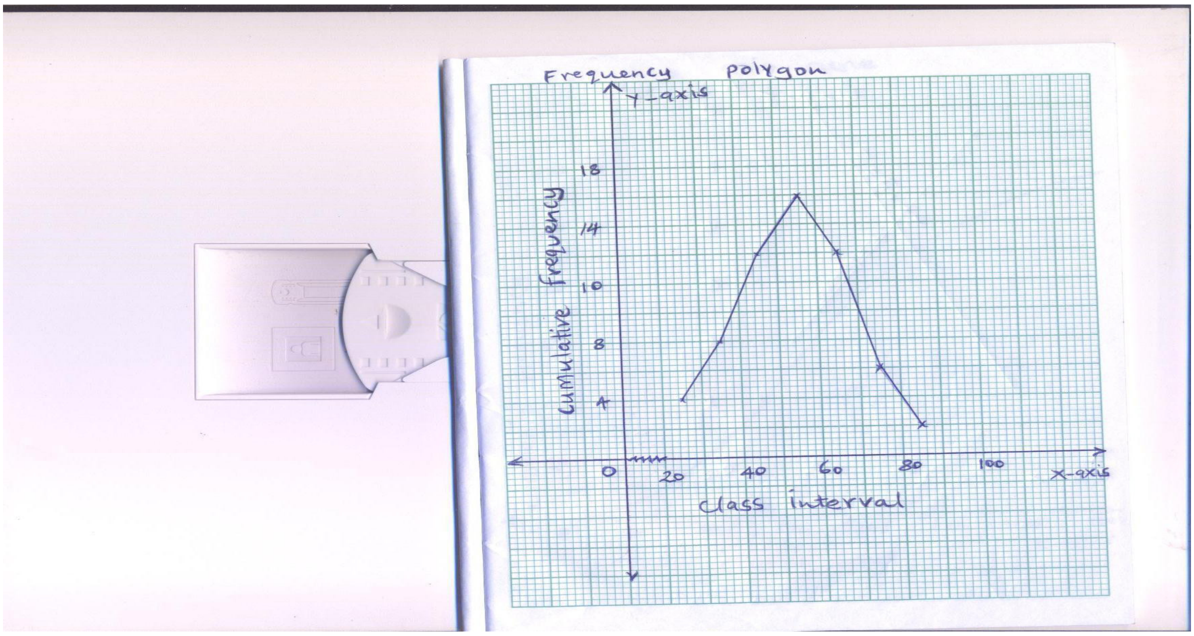
Table 14

| Marks | Midpoint | Frequency (f) |
|---|---|---|
| 20 - 30 | 25 | 4 |
| 30 - 40 | 35 | 8 |
| 40 - 50 | 45 | 12 |
| 50- 60 | 55 | 16 |
| 60- 70 | 65 | 12 |
| 70- 80 | 75 | 6 |
| 80- 90 | 85 | 2 |
| **Total** | | $\Sigma f$= 60 |

Solution:

By taking the steps outlined above, we can easily draw a frequency polygon. Thus,

Figure 18: Frequency Polygon




## Cumulative frequency curve (O-give curve)

A frequency distribution becomes cumulative when the frequency of each class or class-interval is cumulative. The cumulative frequency of a class or class-interval can be obtained by adding the frequency of that class or class-interval to the sum of the frequencies of the preceding classes or class-intervals.

Many a times, we want to know the number of data items or observations which fall below, or, above a certain value. For that reason, there are two types of cumulative frequencies, i.e., less than (from below) cumulative frequency, and more than (from above) cumulative frequencies.

In the less than type, the cumulative frequency of each class or class-interval is obtained by adding the frequencies of the given class and all the preceding classes, when the classes are arranged in the ascending order of the value of the variable. In the more than type of cumulative frequency, the cumulative frequency of each class or class-interval is obtained by adding the frequencies of the given class and those of the succeeding classes.

In order to plot the o-give curve for presenting a given data set, there a number of steps to go through. The simple steps to be followed include:

i) First plot the axes as required
ii) Label the $x$-axis (horizontal axis) with the full range of the data and the $y$-axis (vertical axis) from 0 to and beyond the highest cumulative frequency
iii) Mark or plot the cumulative frequency curve at the *end point* of each class
iv) Join the adjacent points, starting at 0; at the lowest class boundary

### Example:

Use the distribution table 15 given below to plot an o-give curve.

**Table 15**

| **Height (cm)** | 160 - 164 | 165 - 169 | 170 - 174 | 175 - 179 | 180 - 184 | 185 - 189 |
|---|---|---|---|---|---|---|
| **Frequency f** | 16 | 24 | 30 | 22 | 14 | 9 |

**Solution**

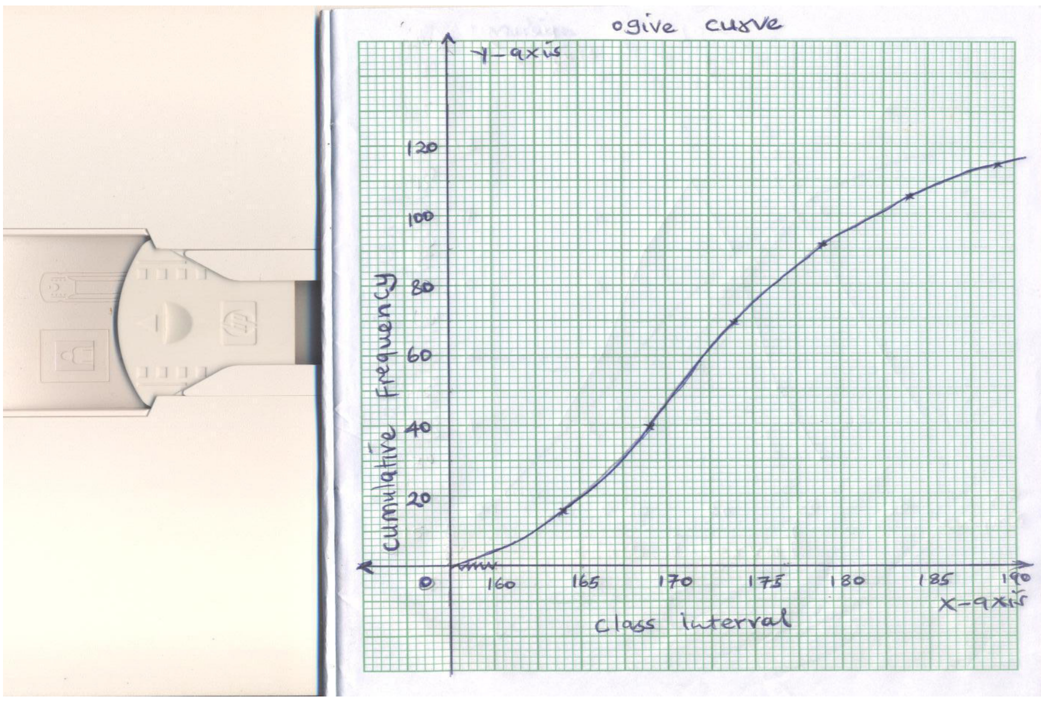
Draw a cumulative frequency table as required

Table 16: Frequency Table

| Height (cm) | Frequency (f) | Cumulative frequency (cf) |
|---|---|---|
| 160 - 164 | 16 | 16 |
| 165 - 169 | 24 | 40 |
| 170 - 174 | 30 | 70 |
| 175 - 179 | 22 | 92 |
| 180 - 184 | 14 | 106 |
| 185 - 189 | 9 | 115 |
| | | |

From the cumulative frequency table, draw/plot the o-give curve as shown below:

Figure 1: Cumulative frequency curve (O-give curve)



## 7.8 References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## 8.0 USE BASIC FUNCTIONS OF CALCULATOR

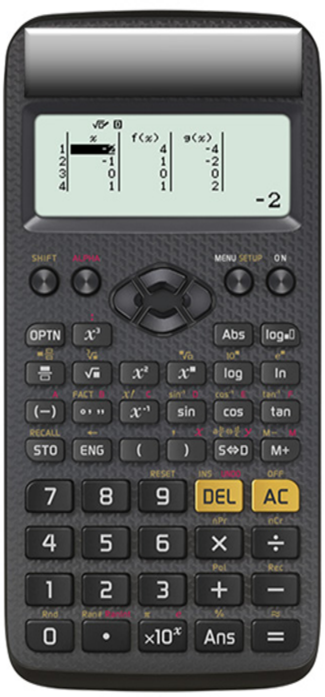
### 8.1 Specific Learning Outcomes

At the end of the topic, the trainee should be able to:

i. Identify and use keys for basic functions on a calculator
ii. Calculate using whole numbers, money and routine decimals and percentages
iii. Calculate with routine fractions and percentages
iv. Apply order of operations to solve multi-step calculations
v. Interpret display and record result
vi. Make estimations to check reasonableness of problem-solving process, outcome and its appropriateness to the context and task
vii. Use formal and informal mathematical language and appropriate symbolism and conventions to communicate the result of the task

### 8.2 General Layout of the Calculator

Figure 20: Scientific Calculator



**Basic Keys**

**a) Display Screen**

This part displays all the information or operations that are being keyed in. The display screen lights up when the calculator is turned on. The screen in addition has display indicators to indicate the mode being used.

**b) On Key**

This key is used to turn on the scientific calculator.

**c) Number Keys**

The number keys are located on the left-hand corner of the calculator and they are arranged from left to right; starting from the lower bottom to the top. The numbers start from 0,1, through to 9. The decimal point (.) is also included under the number keys to facilitate in the writing of decimal numbers.

**d) Basic Operations and Equals Keys**

These keys are on the lower right hand side of the calculator. These keys include the addition (+), subtraction (-), multiplication (x) and the division (÷).

Below the operations keys there is the equals key (=). This key is used to generate the output or give an answer.

**e) All Clear and Delete Key**

The All clear (AC) key is used to clear the screen (everything that has been displayed or you would probably wish to start keying in afresh). It also turns off. On the other hand, the delete (DEL) key is used to erase any input or information that has been keyed in by mistake.

**NOTE:** While the calculator is still on, pressing the SHIFT key and then AC, switches it off.

**f) Cursor Control Button**

These keys control the movement of the cursor while carrying out operations to the required position(s) on the screen display.

**g) Functions keys**

These keys consist of square roots, reciprocals, fractions, logarithms and trigonometric keys among others. The functions keys and their corresponding usage are explained in the subsequent sections which show how they are applied in mathematical calculations.

**h) Shift Key**

The shift key is used for accessing other function keys that are either opposite or second function. It activates the highlighted yellow functions.

### i) Alpha Key

This function allows numerical values stored in the calculator memories to be used within calculations and are accessed by pressing the ALPHA button before the appropriate key. The ALPHA key allows one to key in or name variables in different formulae. When the ALPHA button is pressed, the symbol 'A' is displayed at the top of the calculator. The ALPHA key activates the function in readiness for application. It activates the highlighted light red functions.

### j) Menu Setup Key

The menu setup key displays the mode the calculator is operating on. Normally, the calculator shows 3 modes; that is:

Mode 1 - Calculate
Mode 2 - Statistics
Mode 3 - Table

To access the menu setup function, press the menu setup key next to the ON key. If you need to initialize the calculator to its factory setting in the calculation mode, press SHIFT, 9, 3, On the screen display, you'll see the statement 'Reset OK?', followed by 'Initialize All' with two options: [=]: Yes, and [AC]: Cancel. When you press [=]: Yes, a word is displayed 'Reset!' followed by 'Initialize All' and then 'Press [AC] key'. Pressing the AC key, the calculator will be initialized.

**NOTE:** The importance of initializing the calculator will cancel all previous setups which might have been already activated.

### Use of the Cursor

Moving the cursor from one point or term to another when keying in different operations is important. For instance, when adding square roots, fractions and exponents, it is important to move from one term to another.

For example, if you click 3^2+5 is different from 3^2 move cursor right + 5

## 8.3 Basic Operations on Scientific Calculator

Basic calculations are entered into the calculator in exactly the same order as they are written on paper or a book (Natural Textbook Display). The calculator displays the calculation that you enter. When you press the equals key (=), the answer is displayed at the bottom right hand side of the screen.

### i) Addition

Sample item on using the calculator
Add 346 + 209 =

Start by pressing number keys 3, 4 and 6, then press the operation key plus (+) followed by number keys 2, 0 and 9. Then press the equals key (=).
The calculator will display the operation 346 + 209 = and the answer as 555 on the screen.

### ii) Subtraction

Sample item on using the calculator
Subtract 783 - 257 =

Start by pressing the number keys 7, 8 and 3, then press the operation key minus (-) followed by the number keys 2, 5 and 7. Then press the equals (=) key.

The calculator will display the operation 783 - 257 = and the answer as 526 on the screen.

### iii) Multiplication

Sample item on using the calculator
Multiply 316 x 42 =

Start by pressing the number keys 3, 1 and 6, then press the operation key multiplication (x) followed by number keys 4 and 2. Then press the equals (=) key.

The calculator will display the operation 316 x 42 = and the answer as 13272 on the screen.

### iv) Division

Sample item on using the calculator
Divide 13431÷ 37 =

Start by pressing the number keys 1, 3, 4, 3 and 1, then press operation key division (÷) followed by the number keys 3 and 7. Then press the equals (=) key.

The calculator will display the operation 13431÷ 37 = and the answer as 363 on the screen.

### v) Combined operation

Sample item on using the calculator

#### Example 1

Work out 8 (75 ÷ 5) + 306 =

Start by pressing the number keys 8, opening bracket, then 7, 5, division key, 5 and closing brackets. Then press the addition key followed by the number keys 3, 0 and 6. Lastly, press the equals (=) key.

The calculator will display the operation 8(75 ÷ 5) +306 and the answer as 426 on the screen.

#### Example 2 : 2x3-4^3 +6

Start by pressing 2, then multiplication key, followed by 3, then minus key, press 4, power key function, 3, move cursor right press plus then 6 lastly =

### vi) Squares

Sample item on using calculator
Work out $63^2$

Start by pressing number keys 6 and 3 followed by square function. Press equals key. The calculator will display the operation $63^2$ and the answer 3,969 on the screen.

### vii) Square Roots

Sample item on using calculator
Work out √758,641

Start by pressing square root function followed by pressing number keys 7,5,8,6,4 and 1. Press equals key.
The calculator will display the operation √758,641 and the answer 871 on the screen.

### viii) Decimals

Sample item on using calculator
Work out: 64.56 + 172.9

Start by pressing number keys 6, 4, decimal key, 5 and 6 followed by addition key and 1, 7, 2 decimal key and 9. Press equals key.

The calculator will display the operation 64.56 + 172.9 = and the answer 237.46 on the screen.

### ix) Reciprocals

Sample item on using calculator
Find the reciprocal of 125

Start by pressing number keys 1, 2 and 5, then press the reciprocal key followed by the equals key.
The calculator will display the operation 125 = and the answer 0.008 on the screen.

### x) Fractions

Sample item on using calculator
Work out the fraction

$^{21}/_{60} \div {}^{5}/_{12} =$

Start by pressing the fraction key, then press 2 and 1 keys as numerator. Then press the cursor control button on the arrow pointing downwards. Press 6 and 0 keys and then press the arrow key on the cursor control key to move from the denominator followed by the division key. Then press the fraction key followed by 5 keys followed by a cursor control button on an arrow pointing downwards. Press 1 and 2 keys and then press the arrow key on the cursor control key to move from the denominator. Press equals key.

The calculator will display the operation
$^{21}/_{60} \div {}^{5}/_{12} =$
and the answer $^{21}/_{25}$ on the screen.

### xi) Percentages

Sample item on using calculator
Convert into percentage

Start by pressing the fraction key and the number keys 2 and 6. Press the down key on the cursor control button, then press 5 and 0 keys followed by = key. The calculator will display $^{13}/_{25}$. Press SD key the answer is given as 0.52. Then press the multiplication symbol followed by 100, then the = key. The calculator will display 52.

### xii) Cubes

Sample item on using calculator
Work out $28^3$

Start by pressing number keys 28 followed by exponent key function then press 3 click on the cursor to the right to move from the exponent Press equals key.

The calculator will display the operation $28^3$ and the answer 21,952 on the screen.

### xiii) Cube roots

Sample item on using calculator

Determine $\sqrt[3]{4{,}096}$

Activate cubic root function by clicking on shift followed by square root key, then input 4,0,9, and 6

The calculator will display the operation $\sqrt[3]{4{,}096}$ and the answer 16 on the screen.

Example 2:

$\sqrt[3]{27} + \sqrt[3]{64}$

Start by activating the cubic root by clicking on shift then square root function key followed by the keys 2 and 7, to type the second term move the cursor to the right to move out from the radicand, redo the steps again activate cubic root by clicking on shift then square root function key followed by 6 and 4 keys then press the cursor to the right to move out from radicand then press = key

The calculator will show the answer 7

## 8.4 Indices, Logarithms and Antilogarithms

### Introduction to Indices

An index number is a number which is raised to a certain power. The power, also known as the index and it shows how many times you have to multiply the number by itself.

### Example 1

Evaluate $2^6$: . This means that you have to multiply 2 by itself six times, therefore

$2^6 = 2\times2\times2\times2\times2 = 64$.

To work out this from the calculator, one has to key in 2 followed by the button exponent function key "X^"

Followed by 6 then move the cursor to the right to move from the exponent then press = This will give 64 as the answer.

### Example 2

Evaluate $30^5$:

To evaluate this from using the scientific calculator, To evaluate this, key in 30 followed by exponent function key "X^" then 5, then move the cursor to the right and lastly the = sign. (30X^5 =)

This gives 24300000.

### Example 3

Evaluate:

To evaluate this, key in ( , followed by -3, followed by ), followed by "X^" then 5 move cursor to the right and lastly the = sign i.e. (-3)X^5 =

This gives -243.

### Example 4

Evaluate $3^{4.5}$:

To evaluate this, key in 3 followed by exponent function key "X^" then 4.5 move the cursor right and lastly the = sign i.e. 3X^4.5 =

This gives 140.2961154…

If you need to round your answer to the nearest hundredths press shift, then menu, press 3 (Number Format) then press 1 (FIX) press 2 then press = the screen will show the answer 140.30

To remove the rounding set-up, initialize the calculator by clicking on SHIFT, 9, 3, = AC.

### Example 5

Evaluate: $(3)^{-2}$

To evaluate this, key in, ( , followed by 3, followed by ), followed by exponent function key "X^" then -2 move cursor to right and lastly the = sign i.e. (3)X-2 =

This gives ¹⁄₉. to find answer in decimal click SD key

### Example 6

Evaluate: $3^8 \div 3^3$

To evaluate this, key in 3 followed by exponent function key "X^" then 8, then the right cursor key , followed by ÷ followed by 3, followed by exponent function key "X^" then 3, followed by the right cursor key and lastly the = sign. (38X^8⇒ 3X^3⇒=)

The answer to this is 243.

### Laws of Indices

There are a number of important rules of index numbers:

1) $= x^a \times x^b = x^{a+b}$
2) $= x^a \div x^b = x^{a-b}$

**NOTE:** Any number raised to power Zero "0" is one i.e.
= 1
Solved examples:
Evaluate: Evaluate: $5^7x5^3$ ( 5 power 7 multiplied by 5 power 3)
This is the same as $5^{7+3}$ ( 5 power the sum of 7 and 3)
This gives ( this gives 5 power 10)
To evaluate this, key in 5 followed by exponent function key "X^" then 10 and lastly the = sign. (5X^10 =)
This gives 9765625.

Evaluate: $7^6 \div 7^2$

$$7^6 \div 7^2 = 7^{6-2}$$
$$= 7^4$$
$$= 2401$$

### Powers of 10

Evaluate:
To evaluate this, key in 10, followed by exponent function key "X^" followed 2 and lastly the = sign (10X^2 =) 10 power 2
This gives 100
Evaluate:
To evaluate this, key in 10, followed by exponent function key "X^" followed 6 and lastly the = sign (10X^6 =) 10 power 6
This gives 1000000.

### Logarithms

Consider the following powers of 3:

$3^0=1$
$3^1 =^1$
$3^2 =9$
$3^3 =27$
$3^4 =81$

The indices 0, 1, 2, 3, 4, ...are called the **logarithms** of the corresponding Numbers to **base 3.**

### Example 1:

Logarithm of 9 to base 3 is 2 i.e. $Log_3$ 9 = 2

### Example 2:

Logarithm of 81 to base 3 is 4 i.e. $Log_3$ 81 = 4
Generally, the expression
$a^m$=n is written as $Log_a$ n = m
$a^m$=n is the **index notation** and $Log_a$ n = m is the **logarithmic notation**.

### Common Logarithms of integers greater than 1

1, 10, 100 and 1000 can be expressed as powers of 10 as follows:
1 $=10^0$
10 = 1
100 = 2 and
1000 = 3
The indices 0, 1, 2 and 3 are called the logarithms to base 10 of 1, 10, 100 and 1000 respectively. Since the base is 10, they are referred to as **common logarithms**.
In this video programme, we will refer to logarithms to base 10 as logarithms.
The logarithm of a number, for example 100 to base 10 is written as $Log_{10}$ 100 = 2.

### Using the Scientific Calculator to solve logarithmic functions

Evaluate:

a) $Log_{10}$ 379
To evaluate this, press the key "Log 'followed by the number 379 and then the equality sign (log 379 =).
This is equal to 2.5786
b) $Log_{10}$ 5280
To evaluate this, press the key "Log 'followed by the number 5280 and then the equality sign (log 5280 =).
This is equal to 3.7226
c) $Log_{10}$ 4500
To evaluate this, press the key "Log 'followed by the number 4500 and then the equality sign (log 4500 =).
This is equal to 3.6532

**Note:**
To find the antilogarithm of a number, X, press the shift key, followed by the "Log" button and lastly the number whose antilogarithm is being found then press = (SHIFT Log X =).

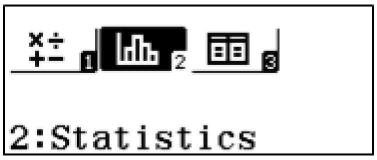
## 8.5 Statistics

There are three main menus in scientific calculator, that is calculation modes. These are:

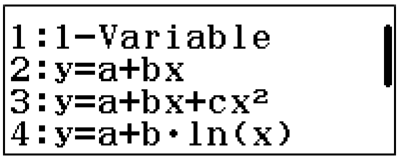
1. Calculate- for general calculations
2. Statistics- for Statistical calculations
3. Table- for generating number tables

Our focus is on Statistics.
To set the calculator in statistics mode, initialize by clicking shift 9 3 = AC. to ensure all the statistics memories are empty

Press Menu button followed by 2, the display screen will show.

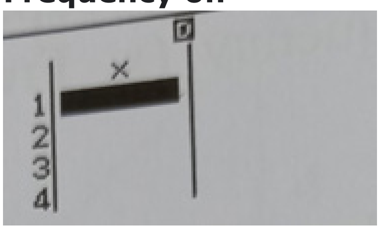
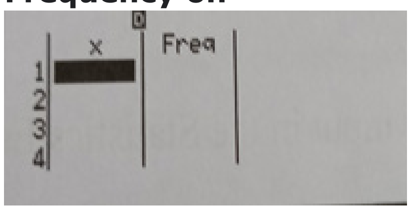


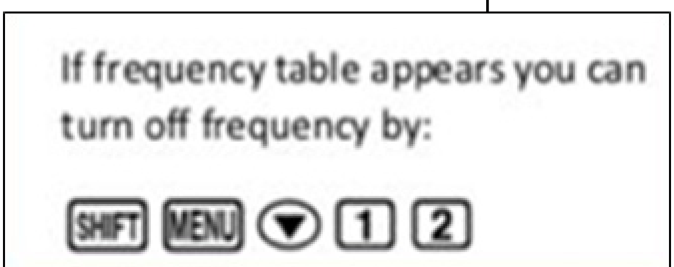

Press 1 for 1 variable statistics, which means single variable.




The screen will display either one column *x* which allows entering data with no frequency or two columns for data with frequency.

Figure 21: Screen Display

| Frequency off | Frequency on |
|---|---|
|  |  |




### To work out statistical values of data without frequency

**Example 1**

Determine the mean of the following data
6, 9, 2, 9, 8, 3, 1, 5.

**Step 1**: From the menu screen select Statistics (Menu 2)
**Step 2**: Select 1- Variable by pressing 1
**Step 3**: A screen will appear showing only one column x input the data as follows
**6=9=2=9=8=3=1=5=**

NB: if the screen shows another column titled freq as explained above, click shift menu scroll down using cursor control button select statistics then press 2 to turn off freq table.
**Step 4**: once finished entering data press **AC**, the screen will clear and Statistics 1- variable appear. To continue press option key select number 2 (1-Variable Calculate) a screen will appear showing all answers of mean, mode, sum of x squared, standard deviation. (*for video show screen appearance for all statistical results*)
The mean for the data is 5.375 (*should show on calculator as the first value*)

**Standard Deviation**
**Calculating the Standard Deviation of a non-frequency data:**

**Example 2**
Find the Standard Deviation of the following numbers correct to one decimal place: 3, 5, 6, 8, 10, 13.
**Step 1:** Press Menu set up key and then select 2 for statistics. Now press 1 for "1-VAR"
**Step 2:** key in the numbers starting with **3 = 5 = 6 = 8 = 10 =13, press [ AC]** to clear the display.
**Step 3:** press option then press 2 a screen will show all answers scroll down using cursor button for standard deviation. (*video show results on calculator display*). S. D= 3.3 to 1 dec. pl.

**Example 3**
**Calculating the Standard Deviation from a Frequency data**

Table 17: Frequency data

| Item | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency | 1 | 3 | 4 | 7 | 4 | 3 |

**Step 1:** Set the calculator to frequency mode. Press Shift key and menu Setup key. Scroll down to the second screen and press **1 for "Statistics, and 1 for "ON"**
**Step 2:** Press Menu set up key and then select 2 for statistics. **Now select 1 for "1-VAR"**
**Step 3:** enter the data from the frequency table, pressing **0 = 1 = 2= 3= 4= 5**
**Step 4:** Using the cursor control button, go to the top of the next column in the table, and key in the frequencies pressing **1 = 3 = 4= 7= 4= 3 and press AC key** to clear display
**Step 5:** press option then press **2 for 1-Variable Calculate,** a screen will show all answers ***(video show all results from calculator)***
**The answer is 1.358.**

**Example 4**
**Mean, variance and standard deviation of an ungrouped data**
The following are the marks scored by 5 students in a Mathematics test. 7, 9, 10, 15, 14. Calculate the mean, variance and the standard deviation using the calculator
Start by pressing the Menu Set up button then press 2 to enter the Statistics mode, press 1 to indicate **I -VAR.** Enter the given data **i.e. 7,9,10,15,14** pressing the equal key after every value, then **press AC key** to clear display. You should have a screen showing only at the bottom Statistics 1-Variable. Press option followed by **2 (1 variable calculate)** the screen will show all answers you may scroll down to see more answers.

**Example 5**
**Mean, variance and standard deviation of an ungrouped data**
The following frequency distribution table 18 represents masses in kilograms of children in a class

Table 18: Frequency Distribution

| Mass of children in kg | 35 | 36 | 38 | 40 | 41 | 42 | 43 |
|---|---|---|---|---|---|---|---|
| No. of children | 4 | 3 | 2 | 2 | 2 | 14 | 8 |

Find the mean, variance and the standard deviation using the calculator

Start by pressing the Menu Set up button then press 2 to enter the Statistics mode, press 1 to indicate I -VAR, which means single variable. To activate the frequency column press Shift key then Menu/set up key. Press down the arrow using the cursor control button and press 1 for Statistics then choose frequency ON. Enter the given data i.e. **35,36, 38, 40, 41, 42, 43 pressing the equal key after every value.** Use cursor button to move the cursor to the top of the second column. Then enter the frequency data **4=3=2=2=2=14=8 then press AC key** to clear the display. Press Option followed by **2, a screen will show the answers of mean, mode, standard deviation and others. (*Video show the calculation results on calculator*)**

**Mean, variance and standard deviation of a grouped data**

To determine the statistical measures for grouped data with frequencies, first work out the mid-points of the classes. Then set the calculator 'frequency on' mode. Enter the data as for ungrouped single variable. All the statistical calculations will be obtained where mean. Variance and standard deviation are included.

Using cursor key scroll down to get other values such as **median, lower quartile (appearing as Q1) and upper quartile (Q3)** among other statistical summaries.

This means once you enter the data correctly, press AC key then Option followed by 2, you get all statistics worked out, select the ones that relate to your question. This makes working with calculator much faster, accurate and reliable.

## 4. Trigonometry

Using the calculator to work out trigonometric problems

Sine, cosine and Tan ratios, Inverse trig ratios.

### Introduction to Trigonometry

Basic Trigonometric functions are entered into the calculator in exactly the same order as they are written on paper or book (Natural Textbook Display). Scientific calculator allows the user to find the trigonometric ratios of angles in degrees, radians or Gradians. Before using the calculator to work out trigonometric ratios, you need to set the mode in degrees. To set the mode in degrees, press shift key followed by menu setup key. The calculator displays four setup options on the screen (1. Input/Output, 2. Angle Unit, 3. Number Format and 4. Fraction Result) (*video show the results as they are on calculator).*

Press 2 key to select Angle Unit. The calculator will display three options of mode of angles (1. Degrees, 2. Radians and 3. Gradian). Press 1 key to select Degrees. The degree mode is indicated by the appearance D mode on the top of the screen display. (*video show the results as they are on calculator).*

### Example

Example 1

Determine the amplitude and the period of the function.

y = 3 sin (2x + 40°). (4 marks)

Solution

Using a calculator sine function key, the trainee can create the following table of values. The learner can then easily key in the function 3sin (2x+40°) by substituting the value of x with degrees indicated below.

Table 19: Table of Values

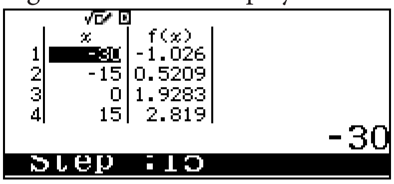
| x | -30 | -15 | 0 | 15 | 22.5 | 30 | 37.5 | 45 | 60 | 75 |
|---|---|---|---|---|---|---|---|---|---|---|
| Y= sin(2x+40°) | -1.03 | 0.52 | 1.93 | 2.82 | 2.99 | 2.95 | 2.72 | 2.3 | 1.02 | -0.52 |

The table of values shows the amplitude (vertical distance between the sinusoidal axis and the maximum or minimum value of the function) to be 2.99      3
By Sketching
Amplitude = 3

### Alternative method

You can use the table function in the calculator to plot the table above. (MENU, 3 then type the function, 3 sin 2 alpha press close brackets key   +40 close brackets followed by selecting the starting point -30 = and ending point 30 =, step 15 then press = = (press equals key twice).

Figure 22: Screen display




When looking for SIN, press the SIN Key found on the 3rd row of the calculator. The word SIN together with the opening bracket automatically appears. Then press the number depicting the angle in degrees followed by the closing bracket (optional). Lastly, press = to get the answer.

### Example 1

Find sin 45°
Using the Scientific Calculator, press sin 45° followed by the = to get the answer. The calculator gives the answer as. The answer displayed is a fraction involving surds instead of a decimal point. To get the decimal point, press SHIFT and then = and thus the answer is given as 0.7071067812. (or click SD if you want to change from fraction to decimal)

### Example 2

Having sin θ = 0.5; find 'theta' in degrees.

### Solution

In your Scientific calculator, Press SHIFT function key followed by the SIN key. The display shows $Sin^{-1}$(. Insert the given decimal point and then press the = in order to get the answer as 30°.

### Cosine

For Cosine, a similar approach like the one used for finding the sine can be followed.

### Example 3

Find cos 60°

### Solution

Press the cos function key (just adjacent to the sin key) followed by the given angle. In this case, it will appear as cos (60°. Then press the = key to give the answer as ½. The fraction can be changed into a decimal point by pressing the S«D function key giving the expression as 0.5.

## 8.6 References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# NOTES

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